lmer for SAS PROC MIXED Users

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1 Introduction

The `lmer` function from the `lme4` library for R is used to fit linear mixed-effects models. It is similar in scope to the SAS procedure `PROC MIXED` described in Littell et al. (1996).

A file on the SAS Institute web site (http://www.sas.com) contains all the data sets in the book and all the SAS programs used in Littell et al. (1996). We have converted the data sets from the tabular representation used for SAS to the `groupedData` objects used by `lmer`. To help users familiar with `SAS PROC MIXED` get up to speed with `lmer` more quickly, we provide transcripts of some `lmer` analyses paralleling the `SAS PROC MIXED` analyses in Littell et al. (1996).

In this paper we highlight some of the similarities and differences of `lmer` analysis and `SAS PROC MIXED` analysis.

2 Similarities between lmer and SAS PROC MIXED

Both `SAS PROC MIXED` and `lmer` can fit linear mixed-effects models expressed in the Laird-Ware formulation. For a single level of grouping Laird and Ware (1982) write the n_i-dimensional response vector $\boldsymbol{y}_i$ for the ith experimental

unit as

$$\begin{aligned} \boldsymbol{y}_i &= \boldsymbol{X}_i\boldsymbol{\beta} + \boldsymbol{Z}_i\boldsymbol{b}_i + \boldsymbol{\epsilon}_i, \quad i = 1, \ldots, M \\ \boldsymbol{b}_i &\sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}), \quad \boldsymbol{\epsilon}_i \sim \mathcal{N}(\boldsymbol{0}, \sigma^2\boldsymbol{I}) \end{aligned} \tag{1}$$

where $\boldsymbol{\beta}$ is the p-dimensional vector of *fixed effects*, $\boldsymbol{b}_i$ is the q-dimensional vector of *random effects*, $\boldsymbol{X}_i$ (of size $n_i \times p$) and $\boldsymbol{Z}_i$ (of size $n_i \times q$) are known fixed-effects and random-effects regressor matrices, and $\boldsymbol{\epsilon}_i$ is the n_i-dimensional *within-group error* vector with a spherical Gaussian distribution. The assumption $\mathrm{Var}(\boldsymbol{\epsilon}_i) = \sigma^2\boldsymbol{I}$ can be relaxed using additional arguments in the model fitting.

The basic specification of the model requires a linear model expression for the fixed effects and a linear model expression for the random effects. In `SAS PROC MIXED` the fixed-effects part is specified in the `model` statement and the random-effects part in the `random` statement. In `lmer` the arguments are called `fixed` and `random`.

Both `SAS PROC MIXED` and `lmer` allow a mixed-effects model to be fit by maximum likelihood (`method = ml` in SAS) or by maximum residual likelihood, sometimes also called restricted maximum likelihood or REML. This is the default criterion in `SAS PROC MIXED`. The default criterion in `lmer` is maximum likelihood. To get REML estimates in `lmer`, set the optional argument `REML=TRUE`.

3 Important differences

One of the most important differences has just been stated but is worth repeating. SAS defaults to REML fits; `lmer` defaults to maximum likelihood fits.

The output from `PROC MIXED` typically includes values of the Akaike Information Criterion (AIC) and Schwartz's Bayesian Criterion (SBC). These are used to compare different models fit to the same data. The output of the `summary` function applied to the object created by `lmer` also produces values of AIC and BIC but the definitions used in `PROC MIXED` and in `lmer` are different. In `lmer` the definitions are such that "smaller is better". In `PROC MIXED` the definitions are such that "bigger is better".

When models are fit by REML, the values of AIC, SBC (or BIC) and the log-likelihood can only be compared between models with exactly the same fixed-effects structure. When models are fit by maximum likelihood these

criteria can be compared between any models fit to the same data. That is, these quality-of-fit criteria can be used to evaluate different fixed-effects specifications or different random-effects specifications or different specifications of both fixed effects and random effects. The greater flexibility of model comparisons when using maximum likelihood is the reason that this is the default criterion in `lmer`.

We encourage developing and testing the model using likelihood ratio tests or the AIC and BIC criteria. Once a form for both the random effects and the fixed effects has been determined, the model can be refit with `REML = TRUE` if the restricted estimates of the variance components are desired.

4 Data manipulation

Both `PROC MIXED` and `lmer` work with data in a tabular form with one row per observation. There are, however, important differences in the internal representations of variables in the data.

In SAS a qualitative factor can be stored either as numerical values or alphanumeric labels. When a factor stored as numerical values is used in `PROC MIXED` it is listed in the `class` statement to indicate that it is a factor. In S this information is stored with the data itself by converting the variable to a factor when it is first stored. If the factor represents an ordered set of levels, it should be converted to an `ordered` factor.

For example the SAS code

```
data animal;
 input trait animal y;
 datalines;
1 1 6
1 2 8
1 3 7
2 1 9
2 2 5
2 3 .
;
```

would require that the `trait` and `animal` variables be specified in a class statement in any model that is fit.

In S these data could be read from a file, say `animal.dat`, and converted to factors by

```
animal <- read.table("animal.dat", header = TRUE)
animal$trait <- as.factor(animal$trait)
animal$animal <- as.factor(animal$animal)
```

In general it is a good idea to check the types of variables in a data frame before working with it. One way of doing this is to apply the function data.class to each variable in turn using the `sapply` function.

```
> sapply(Animal, data.class)
        Sire          Dam AvgDailyGain
    "factor"     "factor"    "numeric"
> str(Animal)
`data.frame':        20 obs. of  3 variables:
 $ Sire        : Factor w/ 5 levels "1","2","3","4",..: 1 1 1 1 2 2 2 2 3 3 .
 $ Dam         : Factor w/ 2 levels "1","2": 1 1 2 2 1 1 2 2 1 1 ...
 $ AvgDailyGain: num  2.24 1.85 2.05 2.41 1.99 1.93 2.72 2.32 2.33 2.68 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 AvgDailyGain ~ 1 | Sire/Dam
  .. .. ..- attr(*, ".Environment")=length 0 <environment>
  ..$ order.groups:List of 2
  .. ..$ Sire: logi TRUE
  .. ..$ Dam : logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       : NULL
  ..$ labels      :List of 1
  .. ..$ AvgDailyGain: chr "Average Daily Weight Gain"
  ..$ units       : list()
```

To make specification of models in `lmer` easier and to make graphic presentations more informative, we recommend converting from a `data.frame` object to a `groupedData` object. This class of objects contains a formula specifying the response, the primary covariate (if there is one) and the grouping factor or factors. The data sets from Littell et al. (1996) have been converted to `groupedData` objects in this directory.

4.1 Unique levels of factors

Designs with nested grouping factors are indicated differently in the two languages. An example of such an experimental design is the semiconductor experiment described in section 2.2 of Littell et al. (1996) where twelve wafers

are assigned to four experimental treatments with three wafers per treatment. The levels for the wafer factor are `1`, `2`, and `3` but the wafer factor is only meaningful within the same level of the treatment factor, `et`. There is nothing associating wafer `1` of the third treatment group with wafer `1` of the first treatment group.

In `SAS` this nesting of factors is denoted by `wafer(et)`. In S the nesting is written with `ET/Wafer` and read "wafer within ET". If both levels of nested factors are to be associated with random effects then this is all you need to know. You would use an expression with a `"/"` in the grouping factor part of the formula for the `groupedData` object. Then the random effects could be specified as

```
random = list( ET = ~ 1, Wafer = ~ 1 )
```

or, equivalently

```
random = ~ 1 | ET/Wafer
```

In this case, however, there would not usually be any random effects associated with the "experimental treatment" or `ET` factor. The only random effects are at the `Wafer` level. It is necessary to create a factor that will have unique levels for each `Wafer` within each level of `ET`. One way to do this is to assign

```
> Semiconductor$Grp <- with(Semiconductor, ET:Wafer)
```

after which we could specify a random effects term of `(1 | Grp)`.

4.2 General approach

As a general approach to importing data into S for mixed-effects analysis you should:

- Create a `data.frame` with one row per observation and one column per variable.
- Use `ordered` or `as.ordered` to explicitly convert any ordered factors to class `ordered`.
- Use `ordered` or `as.ordered` to explicitly convert any ordered factors to class `ordered`.
- If necessary, use `getGroups` to create a factor with unique levels from inner nested factors.

- Specify the formula for the response, the primary covariate and the grouping structure to create a `groupedData` object from the data frame. Labels and units for the response and the primary covariate can also be specified at this time as can `outer` and `inner` factor expressions.

- Plot the data. Plot it several ways. The use of trellis graphics is closely integrated with the `nlme` library. The trellis plots can provide invaluable insight into the structure of the data. Use them.

5 Contrasts

When comparing estimates produced by `SAS PROC MIXED` and by `lmer` one must be careful to consider the contrasts that are used to define the effects of factors. In SAS a model with an intercept and a qualitative factor is defined in terms of the intercept and the indicator variables for all but the last level of the factor. The default behaviour in S is to use the Helmert contrasts for the factor. On a balanced factor these provide a set of orthogonal contrasts. In R the default is the "treatment" contrasts which are almost the same as the SAS parameterization except that they drop the indicator of the first level, not the last level.

When in doubt, check which contrasts are being used with the contrasts function.

To make comparisons easier, you may find it worthwhile to declare

```
> options(contrasts = c(factor = "contr.SAS", ordered = "contr.poly"))
```

at the beginning of your session.

References

Nan M. Laird and James H. Ware. Random-effects models for longitudinal data. *Biometrics*, 38:963–974, 1982.

Ramon C. Littell, George A. Milliken, Walter W. Stroup, and Russell D. Wolfinger. *SAS System for Mixed Models*. SAS Institute, Inc., 1996.

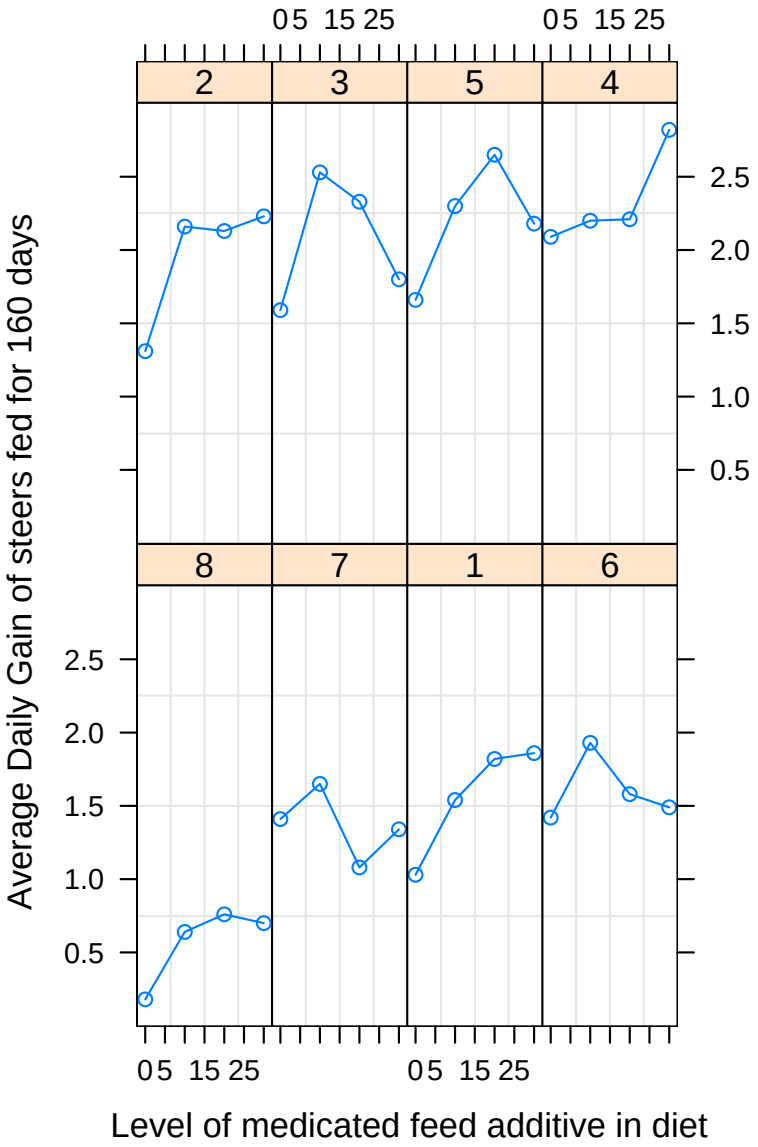

Figure 1: Average daily weight gain

A AvgDailyGain

```
> print(gplot(AvgDailyGain))

> (fm1Adg <- lmer(adg ~ (Treatment - 1) * InitWt + (1 | Block),
+     AvgDailyGain))
Linear mixed-effects model fit by REML
Formula: adg ~ (Treatment - 1) * InitWt + (1 | Block)
   Data: AvgDailyGain
      AIC     BIC    logLik MLdeviance REMLdeviance
 85.32685 99.9842 -32.66342   10.09810     65.32685
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 0.259311 0.50923
 Residual             0.049429 0.22233
# of obs: 32, groups: Block, 8

Fixed effects:
                     Estimate Std. Error DF t value Pr(>|t|)
Treatment0          0.4391368  0.7110882 24  0.6176  0.54268
Treatment10         1.4261185  0.6375459 24  2.2369  0.03485 *
Treatment20         0.4796283  0.5488868 24  0.8738  0.39088
Treatment30         0.2001073  0.7751990 24  0.2581  0.79850
InitWt              0.0044480  0.0020816 24  2.1368  0.04301 *
Treatment0:InitWt  -0.0021543  0.0027863 24 -0.7732  0.44695
Treatment10:InitWt -0.0033651  0.0025148 24 -1.3381  0.19340
Treatment20:InitWt -0.0010823  0.0024875 24 -0.4351  0.66737
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm1Adg)
Analysis of Variance Table
                Df  Sum Sq Mean Sq   Denom F value    Pr(>F)
Treatment        4  5.7248  1.4312 24.0000 28.9543 7.159e-09 ***
InitWt           1  0.5495  0.5495 24.0000 11.1175   0.00277 **
Treatment:InitWt  3  0.1381  0.0460 24.0000  0.9312   0.44088
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> (fm2Adg <- lmer(adg ~ InitWt + Treatment + (1 | Block), AvgDailyGain))
Linear mixed-effects model fit by REML
Formula: adg ~ InitWt + Treatment + (1 | Block)
   Data: AvgDailyGain
```

```
      AIC      BIC    logLik MLdeviance REMLdeviance
 50.33733 60.59748 -18.16866   13.62304     36.33733
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 0.24084  0.49076
 Residual             0.05008  0.22379
# of obs: 32, groups: Block, 8

Fixed effects:
               Estimate  Std. Error DF t value  Pr(>|t|)
(Intercept)  0.80110842  0.35566103 27  2.2524  0.032628 *
InitWt       0.00277971  0.00083335 27  3.3356  0.002486 **
Treatment0  -0.55207364  0.11481306 27 -4.8085 5.096e-05 ***
Treatment10 -0.06856608  0.11896892 27 -0.5763  0.569162
Treatment20 -0.08812909  0.11628776 27 -0.7579  0.455103
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm2Adg)
Analysis of Variance Table
          Df  Sum Sq Mean Sq   Denom F value    Pr(>F)
InitWt     1  0.5146  0.5146 27.0000  10.275 0.0034525 **
Treatment  3  1.5267  0.5089 27.0000  10.162 0.0001185 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> (fm3Adg <- lmer(adg ~ InitWt + Treatment - 1 + (1 | Block),
+     AvgDailyGain))
Linear mixed-effects model fit by REML
Formula: adg ~ InitWt + Treatment - 1 + (1 | Block)
   Data: AvgDailyGain
      AIC      BIC    logLik MLdeviance REMLdeviance
 50.33733 60.59748 -18.16866   13.62304     36.33733
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 0.24084  0.49076
 Residual             0.05008  0.22379
# of obs: 32, groups: Block, 8

Fixed effects:
             Estimate Std. Error DF t value Pr(>|t|)
InitWt      2.7797e-03 8.3335e-04 27  3.3356 0.002486 **
```

```
Treatment0  2.4903e-01 3.7763e-01 27  0.6595 0.515183
Treatment10 7.3254e-01 3.9038e-01 27  1.8765 0.071437 .
Treatment20 7.1298e-01 3.8277e-01 27  1.8627 0.073420 .
Treatment30 8.0111e-01 3.5566e-01 27  2.2524 0.032628 *
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
```

B BIB

```
> print(gplot(BIB))

> (fm1BIB <- lmer(y ~ Treatment * x + (1 | Block), BIB))
Linear mixed-effects model fit by REML
Formula: y ~ Treatment * x + (1 | Block)
   Data: BIB
      AIC     BIC    logLik MLdeviance REMLdeviance
 124.8945 136.675 -52.44723   93.49622     104.8945
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 18.2494  4.2719
 Residual              1.2004  1.0956
# of obs: 24, groups: Block, 8

Fixed effects:
              Estimate Std. Error DF t value  Pr(>|t|)
(Intercept)  22.367853   3.101833 16  7.2112 2.075e-06 ***
Treatment1    4.429485   3.365069 16  1.3163 0.2066152
Treatment2   -0.437371   2.933224 16 -0.1491 0.8833305
Treatment3    6.278627   3.282059 16  1.9130 0.0738148 .
x             0.442547   0.087063 16  5.0831 0.0001107 ***
Treatment1:x -0.223765   0.106083 16 -2.1093 0.0510220 .
Treatment2:x  0.053384   0.097143 16  0.5495 0.5902247
Treatment3:x -0.179177   0.115710 16 -1.5485 0.1410542
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm1BIB)
Analysis of Variance Table
            Df  Sum Sq Mean Sq   Denom  F value    Pr(>F)
Treatment    3  23.447   7.816  16.000   6.5108  0.004367 **
x            1 136.809 136.809  16.000 113.9669 1.098e-08 ***
```

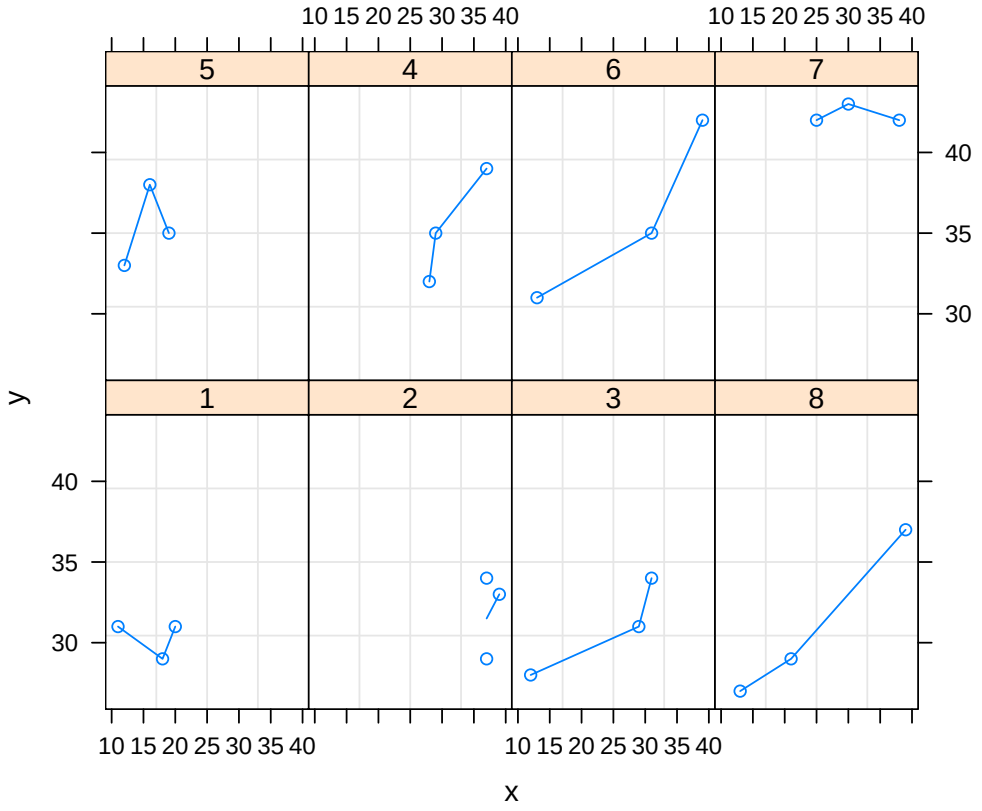

Figure 2: Balanced incomplete block design

```
Treatment:x  3  18.427   6.142  16.000   5.1167  0.011347 *
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> (fm2BIB <- lmer(y ~ Treatment + x:Grp + (1 | Block), BIB))
Linear mixed-effects model fit by REML
Formula: y ~ Treatment + x:Grp + (1 | Block)
   Data: BIB
      AIC      BIC    logLik MLdeviance REMLdeviance
 115.1770 124.6015 -49.58851   94.08883     99.17702
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 18.5255  4.3041
 Residual              1.0378  1.0187
# of obs: 24, groups: Block, 8

Fixed effects:
             Estimate Std. Error DF t value  Pr(>|t|)
(Intercept) 20.945165   2.062297 18 10.1562 7.032e-09 ***
Treatment1   5.341445   1.975705 18  2.7036 0.0145412 *
Treatment2   1.135569   0.713988 18  1.5905 0.1291410
Treatment3   8.181034   1.770100 18  4.6218 0.0002119 ***
x:Grp13      0.239520   0.042964 18  5.5750 2.722e-05 ***
x:Grp24      0.489230   0.044122 18 11.0882 1.781e-09 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm2BIB)
Analysis of Variance Table
          Df  Sum Sq Mean Sq   Denom F value    Pr(>F)
Treatment  3  23.424   7.808  18.000  7.5235  0.001818 **
x:Grp      2 154.733  77.367  18.000 74.5468 1.954e-09 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
```

C Bond

```
> (fm1Bond <- lmer(pressure ~ Metal + (1 | Ingot), Bond))
Linear mixed-effects model fit by REML
Formula: pressure ~ Metal + (1 | Ingot)
   Data: Bond
      AIC      BIC   logLik MLdeviance REMLdeviance
```

```
 117.7902 123.0128 -53.8951   115.7074     107.7902
Random effects:
 Groups   Name        Variance Std.Dev.
 Ingot    (Intercept) 11.448   3.3835
 Residual             10.372   3.2205
# of obs: 21, groups: Ingot, 7

Fixed effects:
            Estimate Std. Error DF t value Pr(>|t|)
(Intercept) 71.10000    1.76552 18 40.2715  < 2e-16 ***
Metalc      -0.91429    1.72143 18 -0.5311  0.60183
Metali       4.80000    1.72143 18  2.7884  0.01213 *
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm1Bond)
Analysis of Variance Table
      Df Sum Sq Mean Sq  Denom F value   Pr(>F)
Metal  2 131.90   65.95  18.00  6.3588 0.008147 **
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
```

D Cultivation

```
> str(Cultivation)
`data.frame':        24 obs. of  4 variables:
 $ Block: Factor w/ 4 levels "1","2","3","4": 1 1 1 1 1 1 2 2 2 2 ...
 $ Cult : Factor w/ 2 levels "a","b": 1 1 1 2 2 2 1 1 1 2 ...
 $ Inoc : Factor w/ 3 levels "con","dea","liv": 1 2 3 1 2 3 1 2 3 1 ...
 $ drywt: num  27.4 29.7 34.5 29.4 32.5 34.4 28.9 28.7 33.4 28.7 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 drywt ~ 1 | Block/Cult
  .. .. ..- attr(*, ".Environment")=length 7 <environment>
  ..$ order.groups:List of 2
  .. ..$ Block: logi TRUE
  .. ..$ Cult : logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       :List of 1
  .. ..$ Cult:Class 'formula' length 2 ~Inoc
  .. .. .. ..- attr(*, ".Environment")=length 7 <environment>
```

```
  ..$ labels      :List of 1
  .. ..$ drywt: chr "Yield"
  ..$ units       : list()
> xtabs(~Block + Cult, Cultivation)
     Cult
Block a b
    1 3 3
    2 3 3
    3 3 3
    4 3 3
> (fm1Cult <- lmer(drywt ~ Inoc * Cult + (1 | Block) + (1 |
+     Cult), Cultivation))
Linear mixed-effects model fit by REML
Formula: drywt ~ Inoc * Cult + (1 | Block) + (1 | Cult)
   Data: Cultivation
      AIC     BIC    logLik MLdeviance REMLdeviance
 86.48742 97.0899 -34.24371   74.94174     68.48742
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 1.20728  1.09876
 Cult     (Intercept) 0.26585  0.51561
 Residual             1.19633  1.09377
# of obs: 24, groups: Block, 4; Cult, 2

Fixed effects:
              Estimate Std. Error DF t value  Pr(>|t|)
(Intercept)   33.52500    0.93100 18 36.0098 < 2.2e-16 ***
Inoccon       -5.50000    0.77341 18 -7.1113 1.256e-06 ***
Inocdea       -2.87500    0.77341 18 -3.7173  0.001577 **
Culta         -0.37500    1.06295 18 -0.3528  0.728343
Inoccon:Culta  0.25000    1.09377 18  0.2286  0.821782
Inocdea:Culta -1.02500    1.09377 18 -0.9371  0.361098
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm1Cult)
Analysis of Variance Table
          Df  Sum Sq Mean Sq   Denom F value   Pr(>F)
Inoc       2 118.176  59.088  18.000 49.3909 4.91e-08 ***
Cult       1   0.656   0.656  18.000  0.5486   0.4684
Inoc:Cult  2   1.826   0.913  18.000  0.7631   0.4807
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
```

```
> (fm2Cult <- lmer(drywt ~ Inoc + Cult + (1 | Block) + (1 |
+     Cult), Cultivation))
Linear mixed-effects model fit by REML
Formula: drywt ~ Inoc + Cult + (1 | Block) + (1 | Cult)
   Data: Cultivation
      AIC      BIC    logLik MLdeviance REMLdeviance
 87.75348 95.99985 -36.87674   76.89738     73.75348
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 1.21284  1.10129
 Cult     (Intercept) 0.25844  0.50837
 Residual             1.16299  1.07842
# of obs: 24, groups: Block, 4; Cult, 2

Fixed effects:
            Estimate Std. Error DF t value  Pr(>|t|)
(Intercept) 33.65417    0.86919 20 38.7192 < 2.2e-16 ***
Inoccon     -5.37500    0.53921 20 -9.9683 3.337e-09 ***
Inocdea     -3.38750    0.53921 20 -6.2823 3.917e-06 ***
Culta       -0.63333    0.84304 20 -0.7512    0.4613
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm2Cult)
Analysis of Variance Table
     Df  Sum Sq Mean Sq   Denom F value    Pr(>F)
Inoc  2 118.176  59.088  20.000 50.8069 1.447e-08 ***
Cult  1   0.656   0.656  20.000  0.5644    0.4613
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> (fm3Cult <- lmer(drywt ~ Inoc + (1 | Block) + (1 | Cult),
+     Cultivation))
Linear mixed-effects model fit by REML
Formula: drywt ~ Inoc + (1 | Block) + (1 | Cult)
   Data: Cultivation
      AIC      BIC    logLik MLdeviance REMLdeviance
 87.67784 94.74616 -37.83892   77.32082     75.67784
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 1.21283  1.10129
 Cult     (Intercept) 0.10364  0.32193
```

```
 Residual             1.16299  1.07842
# of obs: 24, groups: Block, 4; Cult, 2

Fixed effects:
            Estimate Std. Error DF t value  Pr(>|t|)
(Intercept) 33.33750    0.70739 21 47.1275 < 2.2e-16 ***
Inoccon     -5.37500    0.53921 21 -9.9683 2.048e-09 ***
Inocdea     -3.38750    0.53921 21 -6.2823 3.134e-06 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm3Cult)
Analysis of Variance Table
     Df  Sum Sq Mean Sq   Denom F value    Pr(>F)
Inoc  2 118.176  59.088  21.000  50.807 8.988e-09 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
```

E Demand

```
> (fm1Demand <- lmer(log(d) ~ log(y) + log(rd) + log(rt) +
+     log(rs) + (1 | State) + (1 | Year), Demand))
Linear mixed-effects model fit by REML
Formula: log(d) ~ log(y) + log(rd) + log(rt) + log(rs) + (1 | State) +      (
   Data: Demand
       AIC       BIC   logLik MLdeviance REMLdeviance
 -224.1653 -205.4148 120.0826  -260.5218    -240.1653
Random effects:
 Groups   Name        Variance   Std.Dev.
 Year     (Intercept) 0.00026465 0.016268
 State    (Intercept) 0.02948900 0.171724
 Residual             0.00111705 0.033422
# of obs: 77, groups: Year, 11; State, 7

Fixed effects:
             Estimate Std. Error DF t value  Pr(>|t|)
(Intercept) -1.284043   0.723423 72 -1.7750  0.080132 .
log(y)       1.069806   0.103925 72 10.2941 8.553e-16 ***
log(rd)     -0.295342   0.052463 72 -5.6296 3.265e-07 ***
log(rt)      0.039882   0.027889 72  1.4300  0.157034
log(rs)     -0.326739   0.114385 72 -2.8565  0.005595 **
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
```

F HR

```
> (fm1HR <- lmer(HR ~ Time * Drug + baseHR + (Time | Patient),
+     HR))
Linear mixed-effects model fit by REML
Formula: HR ~ Time * Drug + baseHR + (Time | Patient)
   Data: HR
     AIC      BIC    logLik MLdeviance REMLdeviance
 789.607 820.2694 -383.8035   788.1219      767.607
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Patient  (Intercept) 60.630   7.7866
          Time        37.786   6.1470   -0.563
 Residual             24.361   4.9357
# of obs: 120, groups: Patient, 24

Fixed effects:
             Estimate Std. Error  DF t value  Pr(>|t|)
(Intercept)  33.97835   10.28243 113  3.3045  0.001275 **
Time         -3.19704    3.08498 113 -1.0363  0.302263
Druga         3.59915    4.23132 113  0.8506  0.396791
Drugb         7.09121    4.20934 113  1.6846  0.094819 .
baseHR        0.54342    0.11614 113  4.6789 8.058e-06 ***
Time:Druga   -7.50131    4.36282 113 -1.7194  0.088285 .
Time:Drugb   -3.98942    4.36282 113 -0.9144  0.362447
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm1HR)
Analysis of Variance Table
          Df Sum Sq Mean Sq  Denom F value    Pr(>F)
Time       1 379.22  379.22 113.00 15.5665 0.0001387 ***
Drug       2  92.90   46.45 113.00  1.9067 0.1533252
baseHR     1 533.32  533.32 113.00 21.8923 8.058e-06 ***
Time:Drug  2  72.11   36.06 113.00  1.4801 0.2319904
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> (fm3HR <- lmer(HR ~ Time + Drug + baseHR + (Time | Patient),
+     HR))
Linear mixed-effects model fit by REML
Formula: HR ~ Time + Drug + baseHR + (Time | Patient)
   Data: HR
```

```
      AIC      BIC    logLik MLdeviance REMLdeviance
 797.8283 822.9158 -389.9142   791.2093     779.8283
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Patient  (Intercept) 61.560   7.8460
          Time        40.963   6.4002   -0.571
 Residual             24.361   4.9357
# of obs: 120, groups: Patient, 24

Fixed effects:
             Estimate Std. Error  DF t value  Pr(>|t|)
(Intercept)  36.04640   10.19449 115  3.5359 0.0005868 ***
Time         -7.02729    1.81789 115 -3.8656 0.0001839 ***
Druga        -0.45237    3.51456 115 -0.1287 0.8978087
Drugb         4.93648    3.48807 115  1.4152 0.1596980
baseHR        0.54342    0.11615 115  4.6787 7.937e-06 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm3HR)
Analysis of Variance Table
       Df Sum Sq Mean Sq  Denom F value    Pr(>F)
Time    1 364.03  364.03 115.00 14.9431 0.0001839 ***
Drug    2  92.88   46.44 115.00  1.9064 0.1532830
baseHR  1 533.27  533.27 115.00 21.8905 7.937e-06 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> (fm4HR <- lmer(HR ~ Time + baseHR + (Time | Patient), HR))
Linear mixed-effects model fit by REML
Formula: HR ~ Time + baseHR + (Time | Patient)
   Data: HR
      AIC      BIC    logLik MLdeviance REMLdeviance
 805.1481 824.6605 -395.5740   794.2834     791.1481
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Patient  (Intercept) 63.026   7.9389
          Time        40.963   6.4002   -0.553
 Residual             24.361   4.9357
# of obs: 120, groups: Patient, 24

Fixed effects:
```

```
             Estimate Std. Error  DF t value  Pr(>|t|)
(Intercept)  36.93141    9.90143 117  3.7299 0.0002969 ***
Time         -7.02729    1.81789 117 -3.8656 0.0001825 ***
baseHR        0.55078    0.11754 117  4.6857 7.593e-06 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm4HR)
Analysis of Variance Table
       Df Sum Sq Mean Sq  Denom F value    Pr(>F)
Time    1 364.03  364.03 117.00  14.943 0.0001825 ***
baseHR  1 534.87  534.87 117.00  21.956 7.593e-06 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
```

G Mississippi

```
> (fm1Miss <- lmer(y ~ 1 + (1 | influent), Mississippi))
Linear mixed-effects model fit by REML
Formula: y ~ 1 + (1 | influent)
   Data: Mississippi
      AIC      BIC    logLik MLdeviance REMLdeviance
 258.3511 263.1839 -126.1756   256.6398     252.3511
Random effects:
 Groups   Name        Variance Std.Dev.
 influent (Intercept) 63.323   7.9576
 Residual             42.658   6.5313
# of obs: 37, groups: influent, 6

Fixed effects:
            Estimate Std. Error DF t value  Pr(>|t|)
(Intercept)   21.223      3.429 36  6.1892 3.885e-07 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> (fm1MLMiss <- lmer(y ~ 1 + (1 | influent), Mississippi, method = "ML"))
Linear mixed-effects model fit by maximum likelihood
Formula: y ~ 1 + (1 | influent)
   Data: Mississippi
     AIC      BIC    logLik MLdeviance REMLdeviance
 262.557 267.3898 -128.2785    256.557     252.4286
Random effects:
```

```
 Groups   Name        Variance Std.Dev.
 influent (Intercept) 52.679   7.2580
 Residual             43.883   6.6245
# of obs: 37, groups: influent, 6

Fixed effects:
            Estimate Std. Error DF t value  Pr(>|t|)
(Intercept)   21.217      3.122 36   6.796 6.089e-08 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> ranef(fm1MLMiss)
$influent
  (Intercept)
1   0.3097833
2  -6.5772278
3  -3.7862748
4   2.8826711
5  -5.8435209
6  13.0145691

attr(,"varFac")
attr(,"varFac")$influent
, , 1

          [,1]
[1,] 0.1016979

, , 2

          [,1]
[1,] 0.1276643

, , 3

          [,1]
[1,] 0.1714372

, , 4

          [,1]
```

```
[1,] 0.1463477

, , 5

          [,1]
[1,] 0.1714372

, , 6

          [,1]
[1,] 0.1714372


attr(,"stdErr")
[1] 6.534319
attr(,"class")
[1] "lmer.ranef"
attr(,"class")attr(,"package")
[1] "lme4"
> ranef(fm1Miss)
$influent
  (Intercept)
1    0.309286
2   -6.719332
3   -3.897945
4    2.946104
5   -6.012984
6   13.374871

attr(,"varFac")
attr(,"varFac")$influent
, , 1

          [,1]
[1,] 0.1033735

, , 2

         [,1]
[1,] 0.130316
```

```
, , 3

          [,1]
[1,] 0.1762532

, , 4

          [,1]
[1,] 0.1498429

, , 5

          [,1]
[1,] 0.1762532

, , 6

          [,1]
[1,] 0.1762532


attr(,"stdErr")
[1] 6.531317
attr(,"class")
[1] "lmer.ranef"
attr(,"class")attr(,"package")
[1] "lme4"
> VarCorr(fm1Miss)
 Groups   Name        Variance Std.Dev.
 influent (Intercept) 63.323   7.9576
 Residual             42.658   6.5313
> (fm2Miss <- lmer(y ~ Type + (1 | influent), Mississippi))
Linear mixed-effects model fit by REML
Formula: y ~ Type + (1 | influent)
   Data: Mississippi
      AIC      BIC    logLik MLdeviance REMLdeviance
 244.5246 252.5792 -117.2623   247.4686     234.5246
Random effects:
 Groups   Name        Variance Std.Dev.
```

```
 influent (Intercept) 14.970   3.8691
 Residual             42.514   6.5202
# of obs: 37, groups: influent, 6

Fixed effects:
            Estimate Std. Error DF t value  Pr(>|t|)
(Intercept)  36.4000     4.8449 34  7.5131 1.011e-08 ***
Type1       -20.8000     5.9338 34 -3.5054  0.001302 **
Type2       -16.4619     5.5168 34 -2.9840  0.005238 **
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm2Miss)
Analysis of Variance Table
     Df Sum Sq Mean Sq  Denom F value   Pr(>F)
Type  2 541.76  270.88  34.00  6.3716 0.004466 **
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
```

H Multilocation

```
> str(Multilocation)
`data.frame':        108 obs. of  7 variables:
 $ obs     : num  3 4 6 7 9 10 12 16 19 20 ...
 $ Location: Factor w/ 9 levels "A","B","C","D",..: 1 1 1 1 1 1 1 1 1 1 ...
 $ Block   : Factor w/ 3 levels "1","2","3": 1 1 1 1 2 2 2 2 3 3 ...
 $ Trt     : Factor w/ 4 levels "1","2","3","4": 3 4 2 1 2 1 3 4 1 2 ...
 $ Adj     : num  3.16 3.12 3.16 3.25 2.71 ...
 $ Fe      : num  7.10 6.68 6.83 6.53 8.25 ...
 $ Grp     : Factor w/ 27 levels "A/1","A/2","A/3",..: 1 1 1 1 2 2 2 2 3 3 ..
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 Adj ~ 1 | Location/Block
  .. .. ..- attr(*, ".Environment")=length 17 <environment>
  ..$ order.groups:List of 2
  .. ..$ Location: logi TRUE
  .. ..$ Block   : logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       :List of 1
  .. ..$ Block:Class 'formula' length 2 ~Trt
  .. .. .. ..- attr(*, ".Environment")=length 17 <environment>
```

```
  ..$ labels      :List of 1
  .. ..$ Adj: chr "Adjusted yield"
  ..$ units       : list()
> Multilocation$Grp <- with(Multilocation, Block:Location)
> (fm1Mult <- lmer(Adj ~ Location * Trt + (1 | Grp), Multilocation))
Linear mixed-effects model fit by REML
Formula: Adj ~ Location * Trt + (1 | Grp)
   Data: Multilocation
      AIC      BIC    logLik MLdeviance REMLdeviance
 86.64621 188.5672 -5.323106  -87.14598     10.64621
Random effects:
 Groups   Name        Variance  Std.Dev.
 Grp      (Intercept) 0.0056193 0.074962
 Residual             0.0345787 0.185953
# of obs: 108, groups: Grp, 27

Fixed effects:
                Estimate Std. Error DF t value  Pr(>|t|)
(Intercept)     2.359233   0.115755 72 20.3812 < 2.2e-16 ***
LocationA       0.649300   0.163703 72  3.9663 0.0001705 ***
LocationB       0.066433   0.163703 72  0.4058 0.6860811
LocationC       0.545333   0.163703 72  3.3312 0.0013667 **
LocationD       0.374133   0.163703 72  2.2854 0.0252337 *
LocationE       0.550000   0.163703 72  3.3597 0.0012505 **
LocationF       0.998100   0.163703 72  6.0970 4.861e-08 ***
LocationG       0.360567   0.163703 72  2.2026 0.0308276 *
LocationH       1.014033   0.163703 72  6.1943 3.252e-08 ***
Trt1            0.227200   0.151830 72  1.4964 0.1389186
Trt2           -0.001400   0.151830 72 -0.0092 0.9926685
Trt3            0.423233   0.151830 72  2.7875 0.0067874 **
LocationA:Trt1 -0.188533   0.214721 72 -0.8780 0.3828425
LocationB:Trt1 -0.275233   0.214721 72 -1.2818 0.2040178
LocationC:Trt1 -0.040000   0.214721 72 -0.1863 0.8527423
LocationD:Trt1 -0.535133   0.214721 72 -2.4922 0.0149969 *
LocationE:Trt1 -0.262967   0.214721 72 -1.2247 0.2246830
LocationF:Trt1 -0.271533   0.214721 72 -1.2646 0.2100968
LocationG:Trt1  0.203233   0.214721 72  0.9465 0.3470587
LocationH:Trt1 -0.149533   0.214721 72 -0.6964 0.4884150
LocationA:Trt2 -0.093467   0.214721 72 -0.4353 0.6646509
LocationB:Trt2 -0.322733   0.214721 72 -1.5030 0.1372028
```

```
LocationC:Trt2  0.089600   0.214721 72  0.4173 0.6777105
LocationD:Trt2 -0.296933   0.214721 72 -1.3829 0.1709748
LocationE:Trt2 -0.306933   0.214721 72 -1.4295 0.1571983
LocationF:Trt2 -0.309933   0.214721 72 -1.4434 0.1532374
LocationG:Trt2 -0.108600   0.214721 72 -0.5058 0.6145606
LocationH:Trt2 -0.330600   0.214721 72 -1.5397 0.1280231
LocationA:Trt3 -0.402467   0.214721 72 -1.8744 0.0649358 .
LocationB:Trt3 -0.565500   0.214721 72 -2.6337 0.0103329 *
LocationC:Trt3 -0.122467   0.214721 72 -0.5704 0.5702135
LocationD:Trt3 -0.548400   0.214721 72 -2.5540 0.0127654 *
LocationE:Trt3 -0.328633   0.214721 72 -1.5305 0.1302711
LocationF:Trt3 -0.462567   0.214721 72 -2.1543 0.0345659 *
LocationG:Trt3 -0.252967   0.214721 72 -1.1781 0.2426279
LocationH:Trt3 -0.372033   0.214721 72 -1.7326 0.0874414 .
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm1Mult)
Analysis of Variance Table
            Df Sum Sq Mean Sq  Denom F value    Pr(>F)
Location     8  6.947   0.868 72.000 25.1147 < 2.2e-16 ***
Trt          3  1.222   0.407 72.000 11.7774 2.307e-06 ***
Location:Trt 24  0.997   0.042 72.000  1.2008    0.2710
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> (fm2Mult <- lmer(Adj ~ Location + Trt + (1 | Grp), Multilocation))
Linear mixed-effects model fit by REML
Formula: Adj ~ Location + Trt + (1 | Grp)
   Data: Multilocation
      AIC      BIC   logLik MLdeviance REMLdeviance
 21.99894 59.54877 3.000531  -51.21968    -6.001063
Random effects:
 Groups   Name        Variance  Std.Dev.
 Grp      (Intercept) 0.0050851 0.07131
 Residual             0.0367154 0.19161
# of obs: 108, groups: Grp, 27

Fixed effects:
             Estimate Std. Error DF t value  Pr(>|t|)
(Intercept)  2.532965   0.075990 96 33.3327 < 2.2e-16 ***
LocationA    0.478183   0.097516 96  4.9037 3.828e-06 ***
```

```
LocationB   -0.224433   0.097516 96 -2.3015 0.0235251 *
LocationC    0.527117   0.097516 96  5.4055 4.710e-07 ***
LocationD    0.029017   0.097516 96  0.2976 0.7666828
LocationE    0.325367   0.097516 96  3.3366 0.0012075 **
LocationF    0.737092   0.097516 96  7.5587 2.411e-11 ***
LocationG    0.320983   0.097516 96  3.2916 0.0013947 **
LocationH    0.800992   0.097516 96  8.2140 9.996e-13 ***
Trt1         0.058344   0.052150 96  1.1188 0.2660283
Trt2        -0.188022   0.052150 96 -3.6054 0.0004966 ***
Trt3         0.083785   0.052150 96  1.6066 0.1114247
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> (fm3Mult <- lmer(Adj ~ Location + (1 | Grp), Multilocation))
Linear mixed-effects model fit by REML
Formula: Adj ~ Location + (1 | Grp)
   Data: Multilocation
      AIC      BIC    logLik MLdeviance REMLdeviance
 31.82048 61.32393 -4.910242  -22.17353     9.820484
Random effects:
 Groups   Name        Variance  Std.Dev.
 Grp      (Intercept) 0.0016543 0.040673
 Residual             0.0504389 0.224586
# of obs: 108, groups: Grp, 27

Fixed effects:
             Estimate Std. Error DF t value  Pr(>|t|)
(Intercept)  2.521492   0.068954 99 36.5677 < 2.2e-16 ***
LocationA    0.478183   0.097516 99  4.9037 3.689e-06 ***
LocationB   -0.224433   0.097516 99 -2.3015  0.023459 *
LocationC    0.527117   0.097516 99  5.4055 4.477e-07 ***
LocationD    0.029017   0.097516 99  0.2976  0.766663
LocationE    0.325367   0.097516 99  3.3366  0.001195 **
LocationF    0.737092   0.097516 99  7.5587 2.089e-11 ***
LocationG    0.320983   0.097516 99  3.2916  0.001381 **
LocationH    0.800992   0.097516 99  8.2140 8.335e-13 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> (fm4Mult <- lmer(Adj ~ Trt + (1 | Grp), Multilocation))
Linear mixed-effects model fit by REML
Formula: Adj ~ Trt + (1 | Grp)
```

```
   Data: Multilocation
      AIC     BIC    logLik MLdeviance REMLdeviance
 43.50571 59.5985 -15.75285   14.95111     31.50571
Random effects:
 Groups   Name        Variance Std.Dev.
 Grp      (Intercept) 0.110922 0.33305
 Residual             0.036715 0.19161
# of obs: 108, groups: Grp, 27

Fixed effects:
             Estimate Std. Error  DF t value  Pr(>|t|)
(Intercept)   2.865667   0.073946 104 38.7533 < 2.2e-16 ***
Trt1          0.058344   0.052150 104  1.1188 0.2658141
Trt2         -0.188022   0.052150 104 -3.6054 0.0004804 ***
Trt3          0.083785   0.052150 104  1.6066 0.1111724
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> (fm5Mult <- lmer(Adj ~ 1 + (1 | Grp), Multilocation))
Linear mixed-effects model fit by REML
Formula: Adj ~ 1 + (1 | Grp)
   Data: Multilocation
      AIC      BIC    logLik MLdeviance REMLdeviance
 53.32725 61.37365 -23.66363   43.74522     47.32725
Random effects:
 Groups   Name        Variance Std.Dev.
 Grp      (Intercept) 0.107492 0.32786
 Residual             0.050439 0.22459
# of obs: 108, groups: Grp, 27

Fixed effects:
             Estimate Std. Error  DF t value  Pr(>|t|)
(Intercept)   2.854194   0.066695 107  42.795 < 2.2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
> anova(fm2Mult)
Analysis of Variance Table
         Df Sum Sq Mean Sq  Denom F value    Pr(>F)
Location  8  7.377   0.922 96.000  25.115 < 2.2e-16 ***
Trt       3  1.222   0.407 96.000  11.092 2.571e-06 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
```

```
> (fm2MultR <- lmer(Adj ~ Trt + (Trt - 1 | Location) + (1 |
+     Block), Multilocation, control = list(msV = 1, niterEM = 200)))
iter    0 value 1.460782
final  value 1.471614
converged
Linear mixed-effects model fit by REML
Formula: Adj ~ Trt + (Trt - 1 | Location) + (1 | Block)
   Data: Multilocation
      AIC      BIC     logLik MLdeviance REMLdeviance
 33.47161 76.38571 -0.7358071  -13.30147     1.471614
Random effects:
 Groups   Name        Variance   Std.Dev.   Corr
 Location Trt1        1.4561e-01 3.8159e-01
          Trt2        1.1026e-01 3.3206e-01 0.992
          Trt3        1.1967e-01 3.4593e-01 0.999 0.996
          Trt4        1.1485e-01 3.3889e-01 0.934 0.972 0.946
 Block    (Intercept) 3.7749e-12 1.9429e-06
 Residual             3.7749e-02 1.9429e-01
# of obs: 108, groups: Location, 9; Block, 3

Fixed effects:
             Estimate Std. Error  DF t value  Pr(>|t|)
(Intercept)   2.865667   0.118991 104 24.0830 < 2.2e-16 ***
Trt1          0.058344   0.069985 104  0.8337  0.406375
Trt2         -0.188022   0.059217 104 -3.1751  0.001971 **
Trt3          0.083785   0.064889 104  1.2912  0.199497
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
```

I PBIB

```
> str(PBIB)
`data.frame':        60 obs. of  3 variables:
 $ response : num  2.4 2.5 2.6 2 2.7 2.8 2.4 2.7 2.6 2.8 ...
 $ Treatment: Factor w/ 15 levels "1","10","11",..: 7 15 1 5 11 13 14 1 2 1 .
 $ Block    : Factor w/ 15 levels "1","10","11",..: 1 1 1 1 8 8 8 8 9 9 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 response ~ Treatment | Block
  .. .. ..- attr(*, ".Environment")=length 24 <environment>
  ..$ order.groups: logi TRUE
```

```
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       : NULL
  ..$ labels      : list()
  ..$ units       : list()
> (fm1PBIB <- lmer(response ~ Treatment + (1 | Block), PBIB))
Linear mixed-effects model fit by REML
Formula: response ~ Treatment + (1 | Block)
   Data: PBIB
      AIC      BIC    logLik MLdeviance REMLdeviance
 85.9849 121.5888 -25.99245   22.82830     51.98489
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 0.046522 0.21569
 Residual             0.085559 0.29250
# of obs: 60, groups: Block, 15

Fixed effects:
             Estimate Std. Error DF t value Pr(>|t|)
(Intercept)  2.8913111  0.1664127 45 17.3743  < 2e-16 ***
Treatment1  -0.0737886  0.2220608 45 -0.3323  0.74121
Treatment10 -0.4002495  0.2220608 45 -1.8024  0.07818 .
Treatment11  0.0073879  0.2220608 45  0.0333  0.97361
Treatment12  0.1615102  0.2220608 45  0.7273  0.47079
Treatment13 -0.2735419  0.2220608 45 -1.2318  0.22441
Treatment14 -0.4000000  0.2272002 45 -1.7606  0.08511 .
Treatment15 -0.0320781  0.2220608 45 -0.1445  0.88579
Treatment2  -0.4859962  0.2220608 45 -2.1886  0.03386 *
Treatment3  -0.4363680  0.2220608 45 -1.9651  0.05560 .
Treatment4  -0.1074808  0.2272002 45 -0.4731  0.63845
Treatment5  -0.0864132  0.2220608 45 -0.3891  0.69901
Treatment6   0.0193828  0.2220608 45  0.0873  0.93083
Treatment7  -0.1023261  0.2220608 45 -0.4608  0.64716
Treatment8  -0.1097056  0.2220608 45 -0.4940  0.62369
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
```

J SIMS

```
> str(SIMS)
```

```
`data.frame':        3691 obs. of  3 variables:
 $ Pretot: num  29 38 31 31 29 23 23 33 30 32 ...
 $ Gain  : num  2 0 6 6 5 9 7 2 1 3 ...
 $ Class : Factor w/ 190 levels "1","10","100",..: 1 1 1 1 1 1 1 1 1 1 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 Gain ~ Pretot | Class
  .. .. ..- attr(*, ".Environment")=length 25 <environment>
  ..$ order.groups: logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       : NULL
  ..$ labels      :List of 2
  .. ..$ Pretot: chr "Sum of pre-test core item scores"
  .. ..$ Gain  : chr "Gain in mathematics achievement score"
  ..$ units       : list()
> (fm1SIMS <- lmer(Gain ~ Pretot + (Pretot | Class), SIMS))
Linear mixed-effects model fit by REML
Formula: Gain ~ Pretot + (Pretot | Class)
   Data: SIMS
      AIC      BIC    logLik MLdeviance REMLdeviance
 22393.16 22430.45 -11190.58   22373.79     22381.16
Random effects:
 Groups   Name        Variance  Std.Dev. Corr
 Class    (Intercept) 13.568605 3.68356
          Pretot       0.010648 0.10319  -0.551
 Residual             22.172141 4.70873
# of obs: 3691, groups: Class, 190

Fixed effects:
               Estimate  Std. Error   DF t value  Pr(>|t|)
(Intercept)    7.068137    0.359603 3689  19.655 < 2.2e-16 ***
Pretot        -0.188670    0.016511 3689 -11.427 < 2.2e-16 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ
```