How to generate new distributions in packages "distr", "distrEx"

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November 5, 2008

Abstract

In this vignette, we give short examples how to produce new distributions in packages "distr" and "distrEx". This vignette refers to package version 2.0.2.

Basically there are three ways to produce new distributions in packages "distr" and "distrEx":

1. automatic generation of single distribution objects by arithmetics and the like
2. using generating functions to produce single distribution objects
3. defining new distribution classes / doing it from scratch

We will give short examples of all three of them.

1 Automatic generation by arithmetics and the like

We have made available quite general arithmetical operations to our distribution objects, generating new image distribution objects automatically. As an example, try

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```
> require(distr)
> N ← Norm(mean = 2, sd = 1.3)
> P ← Pois(lambda = 1.2)
> Z ← 2*N + 3 + P
> Z
```

```
Distribution Object of Class: AbscontDistribution
```

```
> plot(Z, panel.first = grid(), lwd=2)
> p(Z)(0.4)
```

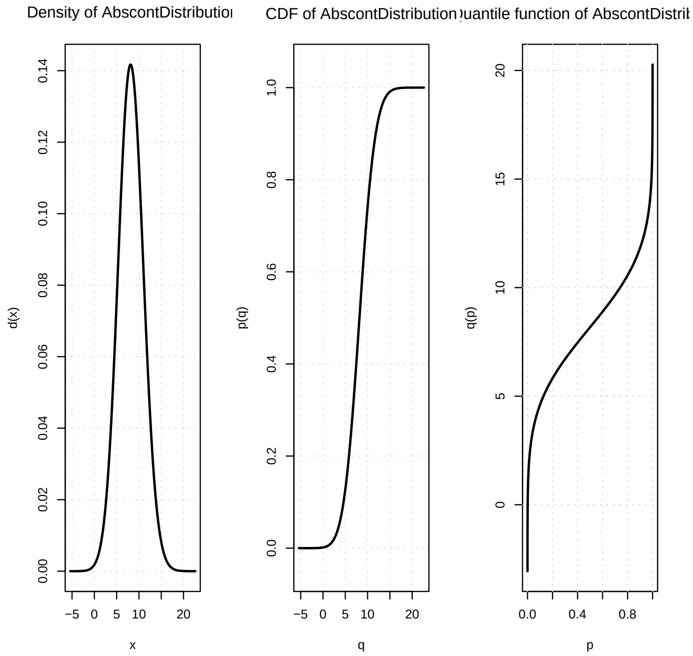
```
[1] 0.002415384
```

```
> q(Z)(0.3)
```

```
[1] 6.70507
```

```
> Zs ← r(Z)(50)
> Zs
```

```
 [1]  9.6383995  8.5482124  9.4126610  5.4995613  6.8803562  7.8664036
 [7]  6.5497598 12.1646822  5.7704260  9.7115487  6.4507198  8.6221548
[13]  7.4119196 10.3743831 10.1295174 10.8241689  6.5987426 10.4875422
[19] 11.4830107  8.2609261  9.8670728  7.8657202  9.8692474 11.8402929
[25] 13.3083332 11.4930382  0.4800001 10.6743194  7.0134726  9.0031499
[31]  5.1196051  6.4768572  7.2975930 11.1205510  7.5198305 12.7816145
[37]  6.8062324 11.5913455 10.1706566  7.3364430  7.3846302  6.5060723
[43] 14.3154418 12.2941129  7.5685204  2.6996885 11.5496830  9.5816526
[49]  8.9335147  2.2137050
```



Comment:
Let `N` an object of class `"Norm"` with parameters `mean=2`, `sd=1.3` and let `P` an object of class `"Pois"` with parameter `lambda=1.2`. Assigning to `Z` the expression `2*N+3+P`, a new distribution object is generated —of class `"AbscontDistribution"` in our case— so that identifying `N`, `P`, `Z` with random variables distributed according to `N`, `P`, `Z`, $\mathcal{L}(\mathtt{Z}) = \mathcal{L}(2*\mathtt{N}+3+\mathtt{P})$, and writing `p(Z)(0.4)` we get $P(Z \leq 0.4)$, `q(Z)(0.3)` the 30%-quantile of `Z`, and with `r(Z)(50)` we generate 50 pseudo random numbers distributed according to `Z`, while the `plot` command generates the above figure.

There a caveats to take care about; for details refer to the (larger) vignette `distr` in package `"distrDoc"`.

2 Using generating functions

If you want to generate a single distribution object (without any particular parameter) generating functions are the method of choice:

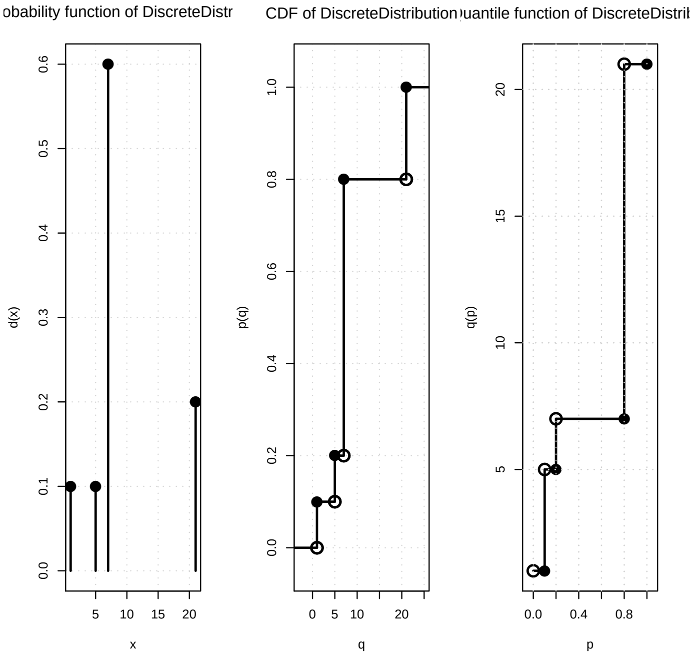
Objects of classes `LatticeDistribution` resp. `DiscreteDistribution`, `AbscontDistribution`, may be generated using the generating functions `LatticeDistribution()` resp. `DiscreteDistribution()` resp. `AbscontDistribution()`; see also the corresponding help.

E.g., to produce a discrete distribution with support $(1, 5, 7, 21)$ with corresponding probabilities $(0.1, 0.1, 0.6, 0.2)$ we may write

```
> D ← DiscreteDistribution(supp = c(1,5,7,21), prob = c(0.1,0.1,0.6,0.2))
> D
```

```
Distribution Object of Class: DiscreteDistribution
```

```
> plot(D, panel.first = grid(), lwd = 2)
```

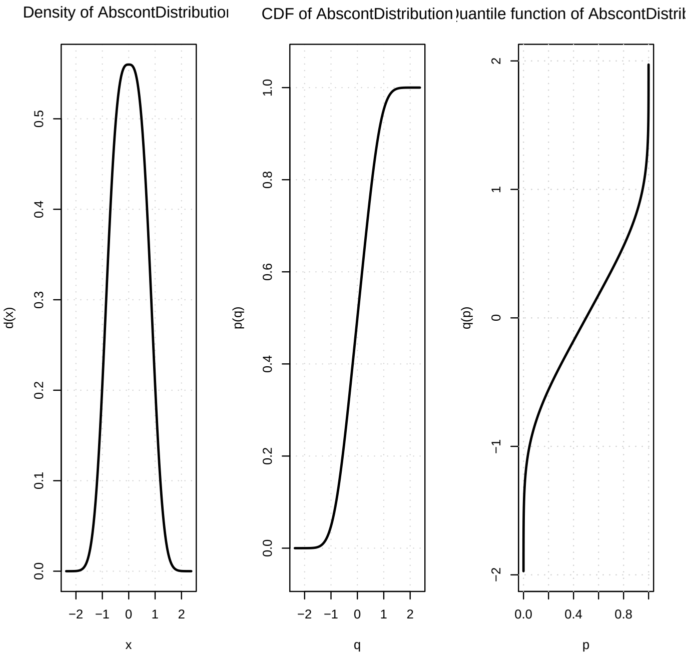


and to generate an absolutely continuos distribution with density proportional to $e^{-|x|^3}$, we write

```
> AC ← AbscontDistribution(d = function(x) exp(-abs(x)^3), withStand = TRUE)
> AC
```

```
Distribution Object of Class: AbscontDistribution
```

```
> plot(AC, panel.first = grid(), lwd = 2)
```



3 Doing it from scratch

If you would like to create new parametric distributions, using already implemented `r`, `d`, `p`, and `q` functions (e.g. implementing additional distributions realized in another CRAN package), you should probably envisage introducing new distribution `S4` (sub-)classes and hence better look at the implementation of some discrete and continuous parametric distribution classes in package `"distr"`.
Hint: download the `.tar.gz` file; extract it to some `temp` folder; look at subdirectories `R` and `man`

The general procedure is as follows

1. introduce a new subclass of class `Parameter`
2. introduce a new subclass of `LatticeDistribution`/`DiscreteDistribution` (if discrete) or of class `AbsContDistribution` (if continuous).
3. define accessor and replacement functions for the "slots" of the parameter (e.g. `"size"` and `"prob"` in the binomial case), possibly with new generics
4. (possibly) define a validity function
5. define a generating function

6. if existing, define particular convolution methods or similar particular methods for this new distribution class

7. create `.Rd` files for the

   - parameter class
   - distribution class

8. if analytic expressions are available, define particular `E`-, `var`-, `skewness`-, and `kurtosis`-methods and if so, also document[1] the corresponding methods in the distribution class `.Rd` file

Let's go through the steps in the example case of the Binomial implementation in packages `"distr"` and `"distrEx"`:

1. in `"distr"`, see source in `R/AllClasses.R`, lines 181–190

```
## Class: BinomParameter
setClass("BinomParameter",
          representation = representation(size = "numeric", prob = "numeric"),
          prototype = prototype(size = 1, prob = 0.5, name =
                      gettext("Parameter␣of␣a␣Binomial␣distribution")
                      ),
          contains = "Parameter"
          )

#-
```

2. in `"distr"`, see source in `R/AllClasses.R`, lines 830–856

```
## Class: binomial distribution
setClass("Binom",
          prototype = prototype(
                      r = function(n){ rbinom(n, size = 1,prob = 0.5) },
                      d = function(x, log = FALSE){
                              dbinom(x, size = 1, prob = 0.5, log = log)
                                          },
                      p = function(q, lower.tail = TRUE, log.p = FALSE ){
                              pbinom(q, size = 1, prob = 0.5,
                                     lower.tail = lower.tail, log.p = log.p)
                                          },
                      q = function(p, lower.tail = TRUE, log.p = FALSE ){
                              qbinom(p, size = 1, prob = 0.5,
```

[1] this is new, because so far, all `E`-, `var`-, `skewness`-, and `kurtosis`-methods for "basic" distributions are documented in the `"distrEx"` documentation to `E`, `var`, . . . , but this would not be operational any longer for new derived classes, possibly defined in other, new packages

```
                                     lower.tail = lower.tail, log.p = log.p)
                                           },
                      img = new("Naturals"),
                      param = new("BinomParameter"),
                      support = 0:1,
                      lattice = new("Lattice",
                                 pivot = 0, width = 1, Length = 2, name =
                                 gettext(
                                   "lattice␣of␣a␣Binomial␣distribution"
                                        )
                                 )
                      ),
          contains = "LatticeDistribution"
          )
```

3. in "distr", see source in R/BinomialDistribution.R, lines 8–15, and 43–53

```
## Access Methods
setMethod("size", "BinomParameter", function(object) object@size)
setMethod("prob", "BinomParameter", function(object) object@prob)
## Replace Methods
setReplaceMethod("size", "BinomParameter",
                 function(object, value){ object@size ← value; object})
setReplaceMethod("prob", "BinomParameter",
                 function(object, value){ object@prob ← value; object})

## wrapped access methods
setMethod("prob", "Binom", function(object) prob(param(object)))
setMethod("size", "Binom", function(object) size(param(object)))
## wrapped replace methods
setMethod("prob←", "Binom",
           function(object, value) new("Binom", prob = value,
                                        size = size(object)))
setMethod("size←", "Binom",
           function(object, value) new("Binom", prob = prob(object),
                                        size = value))
```

and R/AllGenerics, lines 142–145

```
if(!isGeneric("size"))
   setGeneric("size", function(object) standardGeneric("size"))
if(!isGeneric("prob"))
   setGeneric("prob", function(object) standardGeneric("prob"))
```

4. in "distr", see source in R/BinomialDistribution.R, lines 18–32

```
setValidity("BinomParameter", function(object){
  if(length(prob(object)) != 1)
    stop("prob␣has␣to␣be␣a␣numeric␣of␣length␣1")
  if(prob(object) < 0)
```

```
      stop("prob␣has␣to␣be␣in␣[0,1]")
    if(prob(object) > 1)
      stop("prob␣has␣to␣be␣in␣[0,1]")
    if(length(size(object)) != 1)
      stop("size␣has␣to␣be␣a␣numeric␣of␣length␣1")
    if(size(object) < 1)
      stop("size␣has␣to␣be␣a␣natural␣greater␣than␣0")
    if(!identical(floor(size(object)), size(object)))
      stop("size␣has␣to␣be␣a␣natural␣greater␣than␣0")
    else return(TRUE)
  })
```

5. in "distr", see source in R/BinomialDistribution.R, line 41

```
Binom ← function(size = 1,prob = 0.5) new("Binom", size = size, prob = prob)
```

6. in "distr", see source in R/BinomialDistribution.R, lines 54–68

```
## Convolution for two binomial distributions Bin(n1,p1) and Bin(n2,p2)
## Distinguish cases
## p1 == p2 und p1 != p2


setMethod("+", c("Binom","Binom"),
          function(e1,e2){
            newsize ← size(e1) + size(e2)

            if(isTRUE(all.equal(prob(e1),prob(e2))))
               return(new("Binom", prob = prob(e1), size = newsize,
                          .withArith = TRUE))

            return(as(e1, "LatticeDistribution") + e2)
          })
```

7. in "distr", see sources in

- man/BinomParameter-class.Rd

```
\name{BinomParameter-class}
\docType{class}
\alias{BinomParameter-class}
\alias{initialize,BinomParameter-method}

\title{Class "BinomParameter"}
\description{ The parameter of a binomial distribution, used by Binom-class}
\section{Objects from the Class}{
Objects can be created by calls of the form
      \code{new("BinomParameter", prob, size)}.
Usually an object of this class is not needed on its own, it is generated
automatically when an object of the class Binom
is instantiated.
}
\section{Slots}{
  \describe{
    \item{\code{prob}:}{Object of class \code{"numeric"}:
          the probability of a binomial distribution }
    \item{\code{size}:}{Object of class \code{"numeric"}:
         the size of a binomial distribution }
    \item{\code{name}:}{Object of class \code{”character”}:
         a name / comment for the parameters }
  }
}
\section{Extends}{
Class \code{”Parameter”}, directly.
}
\section{Methods}{
  \describe{
    \item{initialize}{\code{signature(.Object = ”BinomParameter”)}:
         initialize method }
    \item{prob}{\code{signature(object = ”BinomParameter”)}: returns the slot
         \code{prob} of the parameter of the distribution }
    \item{prob←}{\code{signature(object = ”BinomParameter”)}: modifies the slot
         \code{prob} of the parameter of the distribution }
    \item{size}{\code{signature(object = ”BinomParameter”)}: returns the slot
         \code{size} of the parameter of the distribution }
    \item{size←}{\code{signature(object = ”BinomParameter”)}: modifies the slot
         \code{size} of the parameter of the distribution}
  }
}

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  }


\seealso{
\code{\link{Binom-class}}
\code{\link{Parameter-class}}
}

\examples{
   W ← new(”BinomParameter”,prob=0.5,size=1)
   size(W) # size of this distribution is 1.
   size(W) ← 2 # size of this distribution is now 2.
}
\keyword{distribution}
\concept{parameter}
\concept{Binomial distribution}
\concept{S4 parameter class}
```

- man/Binom-class.Rd

```
\name{Binom-class}
\docType{class}
\alias{Binom-class}
\alias{Binom}
\alias{initialize,Binom-method}

\title{Class ”Binom” }
\description{The binomial distribution with \code{size} \eqn{= n}, by default
  \eqn{=1}, and
  \code{prob} \eqn{= p}, by default \eqn{=0.5}, has density
  \deqn{p(x) = {n \choose x} {p}^{x} {(1-p)}^{n-x}}{
    p(x) = choose(n,x) p^x (1-p)^(n-x)}
  for \eqn{x = 0, \ldots, n}.

  C.f.\code{\link[stats:Binomial]{rbinom}}
}
\section{Objects from the Class}{
Objects can be created by calls of the form \code{Binom(prob, size)}.
This object is a binomial distribution.
}
\section{Slots}{
  \describe{
    \item{\code{img}:}{Object of class \code{”Naturals”}: The space of the
     image of this distribution has got dimension 1 and the
     name ”Natural Space”. }
    \item{\code{param}:}{Object of class \code{”BinomParameter”}: the parameter
         of this distribution (\code{prob}, \code{size}), declared at its
         instantiation }
    \item{\code{r}:}{Object of class \code{”function”}: generates random
          numbers (calls function \code{rbinom}) }
    \item{\code{d}:}{Object of class \code{”function”}: density function (calls
          function \code{dbinom}) }
    \item{\code{p}:}{Object of class \code{”function”}: cumulative function
          (calls function \code{pbinom}) }
    \item{\code{q}:}{Object of class \code{”function”}: inverse of the
           cumulative function (calls function \code{qbinom}).
    The quantile is defined as the smallest value x such that F(x) >= p, where
            F is the cumulative function. }
    \item{\code{support}:}{Object of class \code{”numeric”}: a (sorted)
            vector containing the support of the discrete density function}
  }
}
\section{Extends}{
Class \code{”DiscreteDistribution”}, directly.\cr
Class \code{”UnivariateDistribution”}, by class \code{”DiscreteDistribution”}.\cr
Class \code{”Distribution”}, by class \code{”DiscreteDistribution”}.
}
\section{Methods}{
  \describe{
    \item{+}{\code{signature(e1 = ”Binom”, e2 = ”Binom”)}: For two binomial
             distributions with equal probabilities the exact convolution
             formula is implemented thereby improving the general numerical
             accuracy.}
    \item{initialize}{\code{signature(.Object = ”Binom”)}: initialize method }
    \item{prob}{\code{signature(object = ”Binom”)}: returns the slot \code{prob}
             of the parameter of the distribution }
    \item{prob←}{\code{signature(object = ”Binom”)}: modifies the slot
             \code{prob} of the parameter of the distribution }
    \item{size}{\code{signature(object = ”Binom”)}: returns the slot \code{size}
             of the parameter of the distribution }
    \item{size←}{\code{signature(object = ”Binom”)}: modifies the slot
             \code{size} of the parameter of the distribution }
  }
}


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  }


\seealso{
\code{\link{BinomParameter-class}}
\code{\link{DiscreteDistribution-class}}
\code{\link{Naturals-class}}
\code{\link[stats:Binomial]{rbinom}}
}
\examples{
    B ← Binom(prob=0.5,size=1) # B is a binomial distribution with prob=0.5 and size=1.
    r(B)(1) # # one random number generated from this distribution, e.g. 1
    d(B)(1) # Density of this distribution is  0.5 for x=1.
    p(B)(0.4) # Probability that x<0.4 is 0.5.
    q(B)(.1) # x=0 is the smallest value x such that p(B)(x)>=0.1.
    size(B) # size of this distribution is 1.
    size(B) ← 2 # size of this distribution is now 2.
    C ← Binom(prob = 0.5, size = 1) # C is a binomial distribution with prob=0.5 and size=1.
    D ← Binom(prob = 0.6, size = 1) # D is a binomial distribution with prob=0.6 and size=1.
    E ← B + C # E is a binomial distribution with prob=0.5 and size=3.
    F ← B + D # F is an object of class LatticeDistribution.
    G ← B + as(D,”DiscreteDistribution”) ## DiscreteDistribution
}
\keyword{distribution}
\concept{discrete distribution}
\concept{lattice distribution}
\concept{Binomial family}
\concept{Binomial distribution}
\concept{S4 distribution class}
\concept{generating function}
```

   - you could have: `man/Binom.Rd` for the generating function; in the Binomial case, documentation is in `Binom-class.Rd`; but in case of the Gumbel distribution, in package `"distrEx"`, there is such an extra `.Rd` file

8. in `"distrEx"`, see sources in

   - `Expectation.R`, lines 309–314

     ```
     setMethod("E", signature(object = "Binom",
                              fun = "missing",
                              cond = "missing"),
         function(object, fun, cond){
             return(size(object)*prob(object))
         })
     ```

   - `Functionals.R`, lines 158–164

     ```
     setMethod("var", signature(x = "Binom"),
         function(x,...){
         if((hasArg(fun))||(hasArg(cond)))
             return(var(as(x,"DiscreteDistribution"),...))
         else
             return(size(x)*prob(x)*(1-prob(x)))
         })
     ```

   - `skewness.R`, lines 59–65

     ```
     setMethod("skewness", signature(x = "Binom"),
         function(x,  ...){
         if((hasArg(fun))||(hasArg(cond)))
            return(skewness(as(x,"DiscreteDistribution"),...))
         else
             return((1-2*prob(x))/sqrt(size(x)*prob(x)*(1-prob(x))))
         })
     ```

   - `kurtosis.R`, lines 69–76

     ```
     setMethod("kurtosis", signature(x = "Binom"),
         function(x,  ...){
         if((hasArg(fun))||(hasArg(cond)))
            return(kurtosis(as(x,"DiscreteDistribution"),...))
         else
             p ← prob(x)
             return((1-6*p*(1-p))/(size(x)*p*(1-p)))
         })
     ```

The procedure will be similar for *any* new class of distributions.

Comment In the classes in package `"distr"` (historically the “oldest” in the development of this project), we still use `initialize` methods; this is no longer needed, if you provide generating functions; for this “more recent” approach, confer the realization of class `Gumbel` in package `"distrEx"`.

4 Help needed / collaboration welcome

You are — as announced on http://distr.r-forge.r-project.org — very welcome to collaborate in this project! See in particular http://distr.r-forge.r-project.org/HOWTO-collaborate.txt

With this you should be able to start working.

References

[1] Kohl M., Ruckdeschel P. and Stabla T. General Purpose Convolution Algorithm for Distributions in S4-Classes by means of FFT. unpublished manual

[2] Ruckdeschel P., Kohl M., Stabla T., and Camphausen F. S4 Classes for Distributions. *R-News*, **6**(2): 10–13. http://CRAN.R-project.org/doc/Rnews/Rnews_2006-2.pdf