# Dipole emission near a planar multilayer stack

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December 16, 2013

## 1 Decay rates

From [RE09] (p. 571), and [NH06] (pp. 335–360), the total decay rate for a dipole perpendicular to the interface is

$$M^{\perp}_{\text{tot}} = 1 + \frac{3}{2}\int_0^{\infty} \Re\left\{\frac{q^3}{\sqrt{1-q^2}} r^p(q)\exp\left(2ik_1d\sqrt{1-q^2}\right)\right\}\mathrm{d}q \tag{1}$$

The integrand diverges as $q \to 1$, it is therefore advantageous to perform the substitution $u := \sqrt{1-q^2}$. In order to maintain a real path of integration, the integral is first split into a radiative region ($0 \leq q \leq 1$, $u := \sqrt{1-q^2} \geq 0$), and an evanescent region ($1 \leq q \leq \infty$, $-iu := \sqrt{q^2-1} \geq 0$). After some algebraic manipulation, we obtain,

$$M^{\perp}_{\text{tot}} = 1 + \frac{3}{2}(I_1 + I_2) \tag{2}$$

where

$$\begin{aligned} I_1 + I_2 = &\int_0^1 \left[1-u^2\right]\cdot\Re\left\{r^p(\sqrt{1-u^2})\exp\left(2idk_1u\right)\right\}\mathrm{d}u \\ &+ \int_0^{\infty}\left[1+u^2\right]\cdot\exp\left(-2dk_1u\right)\cdot\Im\left\{r^p(\sqrt{1+u^2})\right\}\mathrm{d}u \end{aligned} \tag{3}$$

Similarly, for the parallel dipole

$$M^{\parallel}_{\text{tot}} = 1 + \frac{3}{4}\int_0^{\infty}\Re\left\{\left[\frac{r^s(q)}{\sqrt{1-q^2}} - r^p(q)\sqrt{1-q^2}\right]\cdot q\cdot\exp\left(2ik_1d\sqrt{1-q^2}\right)\right\}\mathrm{d}q \tag{4}$$

which can be rewritten as,

$$M_{\text{tot}}^{\parallel} = 1 + \frac{3}{4}\left(I_1^{\parallel} + I_2^{\parallel}\right) \tag{5}$$

where

$$\begin{aligned} I_1^{\parallel} + I_2^{\parallel} = & \int_0^1 \Re\left\{\left[r^s(\sqrt{1-u^2}) - u^2 \cdot r^p(\sqrt{1-u^2})\right]\exp\left(2idk_1u\right)\right\} \mathrm{d}u \\ & + \int_0^\infty \exp\left(-2dk_1u\right) \cdot \Im\left\{r^s(\sqrt{1+u^2}) + u^2 \cdot r^p(\sqrt{1+u^2})\right\} \mathrm{d}u \end{aligned} \tag{6}$$

# References


[NH06] Lukas Novotny and B. Hecht. *Principles of Nano-Optics*. Cambridge Univ Pr, January 2006.

[RE09] Eric Le Ru and Pablo Etchegoin. *Principles of Surface-Enhanced Raman Spectroscopy*. Elsevier, 2009.