

Package ‘mme’

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Description

Fit Gaussian Multinomial mixed-effects models for small area estimation: Model 1, with one random effect in each category of the response variable (Lopez-Vizcaino, E. et al., 2013) <doi:10.1177/1471082X13478873>; Model 2, introducing independent time effect; Model 3, introducing correlated time effect. mme calculates direct and parametric bootstrap MSE estimators (Lopez-Vizcaino, E et al., 2014) <doi:10.1111/rssa.12085>.

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mme-package

Multinomial Mixed Effects Models

Description

The mme package implements three multinomial area level mixed effects models for small area estimation. The first model (Model 1) is based on the area level multinomial mixed model with independent random effects for each category of the response variable (Lopez-Vizcaino et al, 2013). The second model (Model 2) takes advantage from the availability of survey data from different time periods and uses a multinomial mixed model with independent random effects for each category of the response variable and with independent time and domain random effects. The third model (Model 3) is similar to the second one, but with correlated time random effects. To fit the models, we combine the penalized quasi-likelihood (PQL) method, introduced by Breslow and Clayton (1993) for estimating and predicting the fixed and random effects, with the residual maximum likelihood (REML) method for estimating the variance components. In all models the package use two approaches to estimate the mean square error (MSE), first through an analytical expression and second by bootstrap techniques.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13 ,153-178.

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicator under a multinomial mixed model with correlated time and area effects. Submitted for review.

Breslow, N, Clayton, D (1993). Approximate inference in generalized linear mixed models. *Journal of the American Statistical Association*, 88, 9-25.

addtolist	<i>Add items from a list</i>
-----------	------------------------------

Description

This function adds items from a list of dimension $d \times t$, where d is the number of areas and t is the number of times periods.

Usage

```
addtolist(B_d, t, d)
```

Arguments

B_d	a list in each area.
t	number of time periods.
d	number of areas.

Value

B_d a list of dimension d .

See Also

[Fbetaf.it](#), [Fbetaf.ct](#), [modelfit2](#), [modelfit3](#)

Examples

```
k=3 #number of categories for the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata2) # data
mod=2
datar=data.mme(simdata2,k,pp,mod)

##Add the time periods
l=addtolist(datar$X,datar$t,datar$d)
```

addtomatrix	<i>Add rows from a matrix</i>
-------------	-------------------------------

Description

This function adds rows from a matrix of dimension $d*t*(k-1)$ times $d*(k-1)$.

Usage

```
addtomatrix(C2, d, t, k)
```

Arguments

C2	a matrix of dimension $d*t*(k-1)$ times $d*(k-1)$.
d	number of areas.
t	number of time periods.
k	number of categories of the response variable.

Value

C22 a matrix of dimension $d*(k-1)$ times $d*(k-1)$.

See Also

[Fbetaf.it](#), [Fbetaf.ct](#), [modelfit2](#), [modelfit3](#)

Examples

```
k=3 #number of categories of the response variable
d=15 # number of areas
t=2 # number of time periods
mat=matrix(1,d*t*(k-1),d*(k-1)) # a matrix

##Add items in the matrix
mat2=addtomatrix(mat,d,t,k)
```

ci	<i>Standard deviation and p-values of the estimated model parameters</i>
----	--

Description

This function calculates the standard deviations and the p-values of the estimated model parameters. The standard deviations are obtained from the asymptotic Fisher information matrix in the fitting algorithms `modelfit1`, `modelfit2`, `modelfit3`, depending of the current multinomial mixed model.

Usage

```
ci(a, F)
```

Arguments

a	vector with the estimated parameters obtained from <code>modelfit1</code> , <code>modelfit2</code> or <code>modelfit3</code> .
F	inverse of the Fisher Information Matrix obtained from <code>modelfit1</code> , <code>modelfit2</code> or <code>modelfit3</code> .

Value

A list containing the following components.

Std. dev	vector with the standard deviations of the parameters. The parameters are sorted per category.
p. value	vector with the p-values of the parameters for testing $H_0: a=0$.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13, 153-178.

See Also

`modelfit1`, `modelfit2`, `modelfit3`.

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata) #data
mod=1 #Type of model
datar=data.mme(simdata,k,pp,mod)

#Model fit
result=modelfit1(pp,datar$Xk,datar$X,datar$Z,datar$initial,datar$y[,1:(k-1)],
```

```

    datar$n,datar$N)
  beta=result[[8]][,1] #fixed effects
  Fisher=result[[3]] #Fisher information matrix

  ##Standard deviation and p-values
  res=ci(beta,Fisher)

```

 data.mme

Function to generate matrices and the initial values

Description

Based on the input data, this function generates some matrices that are required in subsequent calculations and the initial values obtained from the function [initial.values](#).

Usage

```
data.mme(fi, k, pp, mod)
```

Arguments

fi	input data set with (d x t) rows and 4+k+sum(pp) columns. The first four columns of the data set are, in this order: the area indicator (integer), the time indicator (integer), the sample size of each domain and the population size of each domain. The following k columns are the categories of the response variable. Then, the auxiliary variables: first, the auxiliary variables of the first category, second, the auxiliary variables of the second category, and so on. Examples of input data sets are in simdata , simdata2 and simdata3 .
k	number of categories of the response variable.
pp	vector with the number of auxiliary variables per category.
mod	a number specifying the type of models: 1=multinomial mixed model with one independent random effect in each category of the response variable (Model 1), 2=multinomial mixed model with two independent random effects in each category of the response variable: one domain random effect and another independent time and domain random effect (Model 2) and 3= multinomial model with two independent random effects in each category of the response variable: one domain random effect and another correlated time and domain random effect (Model 3).

Value

A list containing the following components.

n	vector with the area sample sizes.
N	vector with the area population sizes.
Z	design matrix of random effects.

Xk	list of matrices with the auxiliary variables per category. The dimension of the list is the number of domains
X	list of matrices with the auxiliary variables. The dimension of the list is the number of categories of the response variable minus one.
y	matrix with the response variable. The rows are the domains and the columns are the categories of the response variable.
initial	a list with the initial values for the fixed and the random effects obtained from initial.values .

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13, 153-178.

See Also

[initial.values](#), [wmatrix](#), [phi.mult](#), [prmu](#), [Fbetaf](#), [phi.direct](#), [sPhikf](#), [ci](#), [modelfit1](#), [msef](#), [mseb](#)

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata2) #Data
mod=2

##Needed matrix and initial values
datar=data.mme(simdata2,k,pp,mod)
```

Fbetaf	<i>Inverse of the Fisher information matrix of the fixed and random effects in Model 1</i>
--------	--

Description

This function calculates the inverse of the Fisher information matrix of the fixed effects (beta) and the random effects (u) and the score vectors S.beta and S.u, for the model with one independent random effect in each category of the response variable (Model 1). [modelfit1](#) uses the output of this function to estimate the fixed and random effects by the PQL method.

Usage

```
Fbetaf(sigmoid, X, Z, phi, y, mu, u)
```

Arguments

sigmap	a list with the model variance-covariance matrices for each domain.
X	list of matrices with the auxiliary variables obtained from <code>data.mme</code> . The dimension of the list is the number of categories of the response variable.
Z	design matrix of random effects.
phi	vector with the values of the variance components obtained from <code>modelfit1</code> .
y	matrix with the response variable except the reference category. The rows are the domains and the columns are the categories of the response variable minus one.
mu	matrix with the estimated mean of the response variable obtained from <code>prmu</code> .
u	matrix with the values of random effects obtained from <code>modelfit1</code> .

Value

A list containing the following components.

F.beta.beta	the first diagonal element of the inverse of the Fisher information matrix.
F.beta.u	the element out of the diagonal of the inverse of the Fisher information matrix.
F.u.u	the second diagonal element of the inverse of the Fisher information matrix.
S.beta	beta scores.
S.u	u scores.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13 ,153-178.

See Also

`data.mme`, `initial.values`, `wmatrix`, `phi.mult`, `prmu`, `phi.direct`, `sPhikf`, `ci`, `modelfit1`, `msef`, `mseb`.

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata) #data
mod=1 #type of model
datar=data.mme(simdata,k,pp,mod)
initial=datar$initial
mean=prmu(datar$n,datar$Xk,initial$beta.0,initial$u.0)
sigmap=wmatrix(datar$n,mean$estimated.probabilities)

#Inverse of the Fisher information matrix
Fisher=Fbetaf(sigmap,datar$X,datar$Z,initial$phi.0,datar$y[,1:(k-1)],
              mean$mean,initial$u.0)
```

Fbetaf.ct	<i>Inverse of the Fisher information matrix of fixed and random effects in Model 3</i>
-----------	--

Description

This function calculates the score vector S and the inverse of the Fisher information matrix for the fixed (β) and the random effects (u_1, u_2) in Model 3. This model has two independent sets of random effects. The first one contains independent random effects u_{1dk} associated to each category and domain. The second set contains random effects u_{2dkt} associated to each category, domain and time period. Model 3 assumes that the u_{2dk} are AR(1) correlated across time. `modelfit3` uses the output of this function to estimate the fixed and random effect by the PQL method.

Usage

```
Fbetaf.ct(sigmap, X, Z, phi1, phi2, y, mu, u1, u2, rho)
```

Arguments

sigmap	a list with the model variance-covariance matrices for each domain obtained from <code>wmatrix</code> .
X	list of matrices with the auxiliary variables obtained from <code>data.mme</code> . The dimension of the list is the number of categories of the response variable minus one.
Z	design matrix of random effects.
phi1	vector with the values of the variance components for the first random effects obtained from <code>modelfit3</code> .
phi2	vector with the values of the variance components for the second random effects obtained from <code>modelfit3</code> .
y	matrix with the response variable, except the reference category. The rows are the domains and the columns are the categories of the response variable minus one.
mu	matrix with the estimated mean of the response variable.
u1	matrix with the values of the first random effect obtained from <code>modelfit3</code> .
u2	matrix with the values of the second random effect obtained from <code>modelfit3</code> .
rho	vector with the values of the correlation parameter obtained from <code>modelfit3</code> .

Value

A list containing the following components.

F	the inverse of the Fisher information matrix of (β, u_1, u_2).
S	(β, u_1, u_2) score vectors

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicators under a multinomial mixed model with correlated time and area effects. Submitted for review.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult.ct](#), [prmu.time](#), [phi.direct.ct](#), [sPhikf.ct](#), [ci](#), [modelfit3](#), [msef.ct](#), [omega](#), [mseb](#)

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=3 #type of model
data(simdata3)
datar=data.mme(simdata3,k,pp,mod)
initial=datar$initial
mean=prmu.time(datar$n,datar$Xk,initial$beta.0,initial$u1.0,initial$u2.0)
sigmap=wmatrix(datar$n,mean$estimated.probabilities) #variance-covariance

##The inverse of the Fisher information matrix and the score matrix
Fisher.beta=Fbetaf.ct(sigmap,datar$X,datar$Z,initial$phi1.0,initial$phi2.0,
  datar$y[,1:(k-1)],mean$mean,initial$u1.0,initial$u2.0,initial$rho.0)
```

Fbetaf.it

The inverse of the Fisher information matrix of the fixed and random effects for Model 2

Description

This function calculates the score vector S and the inverse of the Fisher information matrix for the fixed (β) and the random effects (u_1 , u_2) in Model 2. This model has two independent sets of random effects. The first one contains independent random effects u_{1dk} associated to each category and domain. The second set contains random effects u_{2dkt} associated to each category, domain and time period. Model 2 assumes that the u_{2dk} are independent across time. [modelfit2](#) uses the output of this function to estimate the fixed and random effect by the PQL method.

Usage

```
Fbetaf.it(sigmap, X, Z, phi1, phi2, y, mu, u1, u2)
```

Arguments

sigmap a list with the model variance-covariance matrices for each domain.

X list of matrices with the auxiliary variables obtained from [data.mme](#). The dimension of the list is the number of categories of the response variable minus one.

Z	design matrix of random effects obtained from data.mme .
phi1	vector with the first variance component obtained from modelfit2 .
phi2	vector with the second variance component obtained from modelfit2 .
y	matrix with the response variable, except the reference category obtained from data.mme . The rows are the domains and the columns are the categories of the response variable minus one.
mu	matrix with the estimated mean of the response variable obtained from prmu.time .
u1	matrix with the values of the first random effect obtained from modelfit2 .
u2	matrix with the values of the second random effect obtained from modelfit2 .

Value

A list containing the following components.

F	the inverse of the Fisher information matrix.
S	(beta, u1, u2) scores

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicator under a multinomial mixed model with correlated time and area effects. Submitted for review.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult.it](#), [prmu.time](#), [phi.direct.it](#), [sPhikf.it](#), [ci](#), [modelfit2](#), [msef.it](#), [mseb](#).

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=2 #Type of model
data(simdata2) #data
datar=data.mme(simdata2,k,pp,mod)
initial=datar$initial
mean=prmu.time(datar$n,datar$Xk,initial$beta.0,initial$u1.0,initial$u2.0)
sigmap=wmatrix(datar$n,mean$estimated.probabilities)

##The inverse of the Fisher information matrix of the fixed effects
Fisher=Fbetaf.it(sigmap,datar$X,datar$Z,initial$phi1.0,initial$phi2.0,
  datar$y[,1:(k-1)],mean$mean,initial$u1.0,initial$u2.0)
```

initial.values	<i>Initial values for fitting algorithm to estimate the fixed and random effects and the variance components</i>
----------------	--

Description

This function sets the initial values. An iterative algorithm fits the multinomial mixed models that requires initial values for the fixed effects, the random effects and the variance components. This initial values are used in `modelfit1`, `modelfit2` and `modelfit3`.

Usage

```
initial.values(d, pp, datar, mod)
```

Arguments

d	number of areas.
pp	vector with the number of auxiliary variables per category.
datar	output of function <code>data.mme</code> .
mod	a number specifying the type of model: 1=multinomial mixed model with one independent random effect for each category of the response variable (Model 1), 2=multinomial mixed model with two independent random effects for each category of the response variable: one domain random effect and another independent time and domain random effect (Model 2) and 3= multinomial mixed model with two independent random effects for each category of the response variable: one domain random effect and another correlated time and domain random effect (Model 3).

Value

A list containing the following components, depending on the chosen model.

beta.0	a list with the initial values for the fixed effects beta per category.
phi.0	vector with the initial values for the variance components phi of Model 1.
phi1.0	vector with the initial values for the variance components phi1 of Model 2 or 3.
phi2.0	vector with the initial values for the variance components phi2 of Model 2 or 3.
u	matrix with the initial values for the random effect for Model 1.
u1.0	matrix with the initial values for the first random effect for Model 2 or 3.
u2.0	matrix with the initial values for the second random effect for Model 2 or 3.
rho.0	vector with the initial values for the correlation parameter for Model 3.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13, 153-178.

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicators under a multinomial mixed model with correlated time and area effects. Submitted for review.

See Also

[data.mme](#), [wmatrix](#), [phi.mult.it](#), [prmu.time](#), [Fbetaf.it](#), [phi.direct.it](#), [sPhikf.it](#), [ci](#), [modelfit2](#), [msef.it](#), [mseb](#)

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata)
D=nrow(simdata)
mod=1 #Type of model
datar=data.mme(simdata,k,pp,mod)

## Initial values for fixed, random effects and variance components
initial=datar$initial
```

mmedata

Create objects of class mmedata

Description

This function creates objects of class mmedata.

Usage

```
mmedata(fi, k, pp)
```

Arguments

fi	input data set with (d X t) rows and 4+k+sum(pp) columns. The first four columns of the data set are, in this order: the area indicator (integer), the time indicator (integer), the sample size of each domain and the population size of each domain. The following k columns are the categories of the response variable. Then, the auxiliary variables: first, the auxiliary variables of the first category, second, the auxiliary variables of the second category, and so on. Examples of input data set are in simdata , simdata2 and simdata3 .
k	number of categories of the response variable.
pp	vector with the number of auxiliary variables per category.

See Also

[modelfit1](#), [modelfit2](#), [modelfit3](#)

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata)
r=mmedata(simdata,k,pp)
```

model

Choose between the three models

Description

This function chooses one of the three models.

Usage

```
model(d, t, pp, Xk, X, Z, initial, y, M, MM, mod)
```

Arguments

d	number of areas.
t	number of time periods.
pp	vector with the number of the auxiliary variables per category.
Xk	list of matrices with the auxiliary variables per category obtained from data.mme . The dimension of the list is the number of domains.
X	list of matrices with the auxiliary variables obtained from data.mme . The dimension of the list is the number of categories of the response variable minus one.
Z	design matrix of random effects obtained from data.mme .
initial	output of the function initial.values .
y	matrix with the response variable obtained from data.mme . The rows are the domains and the columns are the categories of the response variable.
M	vector with the area sample sizes.
MM	vector with the population sample sizes.
mod	a number specifying the type of models: 1=multinomial mixed model with one independent random effect in each category of the response variable (Model 1), 2=multinomial mixed model with two independent random effects in each category of the response variable: one domain random effect and another independent time and domain random effect (Model 2) and 3= multinomial model with two independent random effects in each category of the response variable: one domain random effect and another correlated time and domain random effect (Model 3).

Value

the output of the function `modelfit1`, `modelfit2` or `modelfit3`.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13, 153-178.

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicator under a multinomial mixed model with correlated time and area effects. Submitted for review.

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata) #data
mod=1 #Model 1
datar=data.mme(simdata,k,pp,mod)
result=model(datar$d,datar$t,pp,datar$Xk,datar$X,datar$Z,datar$initial,datar$y[,1:(k-1)],
datar$n,datar$N, mod)
```

modelfit1

Function used to fit Model 1

Description

This function fits the multinomial mixed model with one independent random effect per category of the response variable (Model 1), like in the formulation described in Lopez-Vizcaino et al. (2013). The fitting algorithm combines the penalized quasi-likelihood method (PQL) for estimating and predicting the fixed and random effects with the residual maximum likelihood method (REML) for estimating the variance components. This function uses as initial values the output of the function `initial.values`

Usage

```
modelfit1(pp, Xk, X, Z, initial, y, M, MM)
```

Arguments

pp	vector with the number of the auxiliary variables per category.
Xk	list of matrices with the auxiliary variables per category obtained from <code>data.mme</code> . The dimension of the list is the number of domains.
X	list of matrices with the auxiliary variables obtained from <code>data.mme</code> . The dimension of the list is the number of categories of the response variable minus one.
Z	design matrix of random effects obtained from <code>data.mme</code> .

<code>initial</code>	output of the function <code>initial.values</code> .
<code>y</code>	matrix with the response variable except the reference category obtained from <code>data.mme</code> . The rows are the domains and the columns are the categories of the response variable minus 1.
<code>M</code>	vector with the area sample sizes.
<code>MM</code>	vector with the population sample sizes.

Value

A list containing the following components.

<code>Estimated.probabilities</code>	matrix with the estimated probabilities for the categories of response variable.
<code>Fisher.information.matrix.phi</code>	Fisher information matrix of the random effect.
<code>Fisher.information.matrix.beta</code>	Fisher information matrix of the fixed effect.
<code>u</code>	matrix with the estimated random effects.
<code>mean</code>	matrix with the estimated mean of the response variable.
<code>warning1</code>	0=OK,1=The model could not be fitted.
<code>warning2</code>	0=OK,1=The value of the variance component is negative: the initial value is taken.
<code>beta.Stddev.p.value</code>	matrix with the estimated fixed effects, its standard deviations and its p-values.
<code>phi.Stddev.p.value</code>	matrix with the estimated variance components, its standard deviations and its p-values.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13, 153-178.

See Also

`data.mme`, `initial.values`, `wmatrix`, `phi.mult`, `prmu`, `phi.direct`, `sPhikf`, `ci`, `Fbetaf`, `msef`, `mseb`.

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata) #data
mod=1 #type of model
datar=data.mme(simdata,k,pp,mod)

#Model fit
result=modelfit1(pp,datar$Xk,datar$X,datar$Z,datar$initial,datar$y[,1:(k-1)],
  datar$n,datar$N)
```

 modelfit2

Function to fit Model 2

Description

This function fits the multinomial mixed model with two independent random effects for each category of the response variable: one domain random effect and another independent time and domain random effect (Model 2). The formulation is described in Lopez-Vizcaino et al. (2013). The fitting algorithm combines the penalized quasi-likelihood method (PQL) for estimating and predicting the fixed and random effects, respectively, with the residual maximum likelihood method (REML) for estimating the variance components. This function uses as initial values the output of the function [initial.values](#).

Usage

```
modelfit2(d, t, pp, Xk, X, Z, initial, y, M, MM)
```

Arguments

d	number of areas.
t	number of time periods.
pp	vector with the number of the auxiliary variables per category.
Xk	list of matrices with the auxiliary variables per category obtained from data.mme . The dimension of the list is the number of domains.
X	list of matrices with the auxiliary variables obtained from data.mme . The dimension of the list is the number of categories of the response variable minus one.
Z	design matrix of random effects data.mme .
initial	output of the function initial.values .
y	matrix with the response variable obtained from data.mme , except the reference category. The rows are the domains and the columns are the categories of the response variable minus one.
M	vector with the area sample sizes.
MM	vector with the population sample sizes.

Value

A list containing the following components.

Estimated.probabilities	matrix with the estimated probabilities for the categories of response variable.
Fisher.information.matrix.phi	Fisher information matrix of the variance components.
Fisher.information.matrix.beta	Fisher information matrix of the fixed effects.

u1	matrix with the estimated first random effect.
u2	matrix with the estimated second random effect.
mean	matrix with the estimated mean of response variable.
warning1	0=OK,1=The model could not be fitted.
warning2	0=OK,1=The value of the variance component is negative: the initial value is taken.
beta.Stddev.p.value	matrix with the estimated fixed effects, its standard deviations and its p-values.
phi.Stddev.p.value	matrix with the estimated variance components, its standard deviations and its p-values.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicators under a multinomial mixed model with correlated time and area effects. Submitted for review.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult.it](#), [prmu.time](#), [phi.direct.it](#), [sPhikf.it](#), [ci](#), [Fbetaf.it](#), [msef.it](#), [mseb](#)

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=2 #type of model
data(simdata2) #data
datar=data.mme(simdata2,k,pp,mod)

##Model fit
result=modelfit2(datar$d,datar$t,pp,datar$Xk,datar$X,datar$Z,datar$initial,datar$y[,1:(k-1)],
                datar$n,datar$N)
```

modelfit3

Function used to fit Model 3

Description

This function fits the multinomial mixed model with two independent random effects for each category of the response variable: one domain random effect and another correlated time and domain random effect (Model 3). The formulation is described in Lopez-Vizcaino et al. (2013). The fitting algorithm combine the penalized quasi-likelihood method (PQL) for estimating and predicting the fixed and random effects, respectively, with the residual maximum likelihood method (REML) for estimating the variance components. This function uses as initial values the output of the function [initial.values](#).

Usage

```
modelfit3(d, t, pp, Xk, X, Z, initial, y, M, MM, b)
```

Arguments

d	number of areas.
t	number of time periods.
pp	vector with the number of the auxiliary variables per category.
Xk	list of matrices with the auxiliary variables per category obtained from <code>data.mme</code> . The dimension of the list is the number of domains.
X	list of matrices with the auxiliary variables obtained from <code>data.mme</code> . The dimension of the list is the number of categories of the response variable minus one.
Z	design matrix of random effects obtained from <code>data.mme</code> .
initial	output of the function <code>initial.values</code> .
y	matrix with the response variable obtained from <code>data.mme</code> , except the reference category. The rows are the domains and the columns are the categories of the response variable minus one.
M	vector with the area sample sizes.
MM	vector with the population sample sizes.
b	parameter that indicates the bootstrap.

Value

A list containing the following components.

Estimated.probabilities	matrix with the estimated probabilities for the categories of response variable.
Fisher.information.matrix.phi	Fisher information matrix of phi.
Fisher.information.matrix.beta	Fisher information matrix of beta.
u1	matrix with the estimated first random effect.
u2	matrix with the estimated second random effect.
mean	matrix with the estimated mean of the response variable.
warning1	0=OK,1=The model could not be fitted.
warning2	0=OK,1=The value of the variance component is negative: the initial value is taken.
beta.Stddev.p.value	matrix with the estimated fixed effects, its standard deviations and its p-values.
phi.Stddev.p.value	matrix with the estimated variance components, its standard deviations and its p-values.

rho estimated correlation parameter.
rho.Stddev.p.value matrix with the estimated correlation parameter, its standard deviations and its p-values.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicators under a multinomial mixed model with correlated time and area effects. Submitted for review.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult.ct](#), [prmu.time](#), [phi.direct.ct](#), [sPhikf.ct](#), [omega.ci](#), [Fbetaf.ct](#), [msef.ct](#), [mseb](#)

Examples

```
## Not run:

k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=3 #type of model
data(simdata3) #data
datar=data.mme(simdata3,k,pp,mod)

##Model fit
result=modelfit3(datar$d,datar$t,pp,datar$Xk,datar$X,datar$Z,datar$initial,datar$y[,1:(k-1)],
                datar$n,datar$N,0)
## End(Not run)
```

mseb

Bias and MSE using parametric bootstrap

Description

This function calculates the bias and the mse for the multinomial mixed effects models using parametric bootstrap. Three types of multinomial mixed models are considered, with one independent domain random effect in each category of the response variable (Model 1), with two random effects: the first, with a domain random effect and with independent time and domain random effect (Model 2) and the second, with a domain random effect and with correlated time and domain random effect (Model 3). See details of the parametric bootstrap procedure in Gonzalez-Manteiga et al. (2008) and in Lopez-Vizcaino et al. (2013) for the adaptation to these three models. This function uses the output of [modelfit1](#), [modelfit2](#) or [modelfit3](#), depending of the current multinomial mixed model.

Usage

```
mseb(pp, Xk, X, Z, M, MM, resul, B, mod)
```

Arguments

pp	vector with the number of the auxiliary variables per category.
Xk	list of matrices with the auxiliary variables per category obtained from <code>data.mme</code> . The dimension of the list is the number of domains.
X	list of matrices with the auxiliary variables obtained from <code>data.mme</code> . The dimension of the list is the number of categories of the response variable minus one.
Z	design matrix of random effects obtained from <code>data.mme</code> .
M	vector with the area sample sizes.
MM	vector with the population sample sizes.
resul	output of the function <code>modelfit1</code> , <code>modelfit2</code> or <code>modelfit3</code> .
B	number of bootstrap replications.
mod	a number specifying the type of models: 1=multinomial mixed model with one independent random effect in each category of the response variable (Model 1), 2=multinomial mixed model with two independent random effects in each category of the response variable: one domain random effect and another independent time and domain random effect (Model 2) and 3= multinomial model with two independent random effects in each category of the response variable: one domain random effect and another correlated time and domain random effect (Model 3).

Value

a list containing the following components.

<code>bias.pboot</code>	BIAS of the parametric bootstrap estimator of the mean of the response variable
<code>mse.pboot</code>	MSE of the parametric bootstrap estimator of the mean of the response variable
<code>rmse.pboot</code>	RMSE of the parametric bootstrap estimator of the mean of the response variable

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13 ,153-178.

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicator under a multinomial mixed model with correlated time and area effects. Submitted for review.

Gonzalez-Manteiga, W, Lombardia, MJ, Molina, I, Morales, D, Santamaria, L (2008). Estimation of the mean squared error of predictors of small area linear parameters under a logistic mixed model, *Computational Statistics and Data Analysis*, 51, 2720-2733.

See Also

`data.mme`, `initial.values`, `wmatrix`, `phi.mult`, `phi.mult.it`, `phi.mult.ct`, `prmu`, `prmu.time`, `phi.direct`, `phi.direct.it`, `phi.direct.ct`, `sPhikf`, `sPhikf.it`, `sPhikf.ct`, `modelfit1`, `modelfit2`, `modelfit3`, `omega`, `Fbetaf`, `Fbetaf.it`, `Fbetaf.ct`, `ci`.

Examples

```

k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata)
mod=1 # Type of model
datar=data.mme(simdata,k,pp,mod)
##Model fit
result=modelfit1(pp,datar$Xk,datar$X,datar$Z,datar$initial,datar$y[,1:(k-1)],datar$n,datar$N)

B=1 #Bootstrap iterations
ss=12345 #SEED
set.seed(ss)

##Bootstrap parametric BIAS and MSE
mse.pboot=mseb(pp,datar$Xk,datar$X,datar$Z,datar$n,datar$N,result,B,mod)

```

msef

Analytic MSE for Model 1

Description

This function calculates the analytic MSE for the multinomial mixed model with one independent random effect per category of the response variable (Model 1). See Lopez-Vizcaino et al. (2013), section 4, for details. The formulas of Prasad and Rao (1990) are adapted to Model 1. This function uses the output of [modelfit1](#).

Usage

```
msef(pp, X, Z, resul, MM, M)
```

Arguments

resul	the output of the function modelfit1 .
X	list of matrices with the auxiliary variables obtained from data.mme . The dimension of the list is the number of categories of the response variable minus one.
Z	design matrix of random effects obtained from data.mme .
pp	vector with the number of the auxiliary variables per category.
M	vector with the area sample sizes.
MM	vector with the population sample sizes.

Value

mse is a matrix with the MSE estimator calculated by adapting the explicit formulas of Prasad and Rao (1990).

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13, 153-178.

Prasad, NGN, Rao, JNK (1990). The estimation of the mean squared error of small area estimators. *Journal of the American Statistical Association*, 85, 163-171.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult](#), [prmu](#), [phi.direct](#), [sPhikf](#), [modelfit1](#), [Fbetaf](#), [ci](#), [mseb](#).

Examples

```
require(Matrix)

k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata) #data
mod=1 # type of model
datar=data.mme(simdata,k,pp,mod)
# Model fit
result=modelfit1(pp,datar$Xk,datar$X,datar$Z,datar$initial,datar$y[,1:(k-1)],
                datar$n,datar$N)

#Analytic MSE
mse=msef(pp,datar$X,datar$Z,result,datar$N,datar$n)
```

msef.ct

Analytic MSE for Model 3

Description

This function calculates the analytic MSE for the multinomial mixed model with two independent random effects for each category of the response variable: one random effect associated with the domain and another correlated random effect associated with time and domain (Model 3). See details of the model and the expression of mse in Lopez-Vizcaino et al. (2013). The formulas of Prasad and Rao (1990) are adapted to Model 3. This function uses the output of [modelfit3](#).

Usage

```
msef.ct(p, X, result, M, MM)
```

Arguments

p vector with the number of the auxiliary variables per category.

X list of matrices with the auxiliary variables obtained from [data.mme](#). The dimension of the list is the number of categories of the response variable minus one.

result the output of the function `modelfit3`.
 M vector with the area sample sizes.
 MM vector with the population sample sizes.

Value

`mse.analitic` is a matrix with the MSE estimator calculated by adapting the explicit formulas of Prasad and Rao (1990).

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicators under a multinomial mixed model with correlated time and area effects. Submitted for review.

Prasad, NGN, Rao, JNK (1990). The estimation of the mean squared error of small area estimators. *Journal of the American Statistical Association*, 85, 163-171.

See Also

`data.mme`, `initial.values`, `wmatrix`, `phi.mult.ct`, `prmu.time`, `phi.direct.ct`, `sPhikf.ct`, `modelfit3`, `Fbetaf.ct`, `ci`, `omega`, `mseb`.

Examples

```
## Not run:

k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=3 #type of model
data(simdata3) #data
datar=data.mme(simdata3,k,pp,mod)
##Model fit
result=modelfit3(d,t,pp,datar$Xk,datar$X,datar$Z,datar$initial,datar$y[,1:(k-1)],
                 datar$n,datar$N,0)

##Analytic MSE
msef=msef.ct(pp,datar$X,result,datar$n,datar$N)
## End(Not run)
```

msef.it

Analytic MSE for Model 2

Description

This function calculates the analytic MSE for the multinomial mixed model with two independent random effects for each category of the response variable: one random effect associated with the domain and another independent random effect associated with time and domain (Model 2). See details of the model and the expression of mse in Lopez-Vizcaino et al. (2013). The formulas of Prasad and Rao (1990) are adapted to Model 2. This function uses the output of `modelfit2`.

Usage

```
msef.it(p, X, result, M, MM)
```

Arguments

p	vector with the number of the auxiliary variables per category.
X	list of matrices with the auxiliary variables obtained from data.mme . The dimension of the list is the number of categories of the response variable minus one.
result	the output of the function modelfit2 .
M	vector with the area sample sizes.
MM	vector with the population sample sizes.

Value

mse.analitic is a matrix with the MSE estimator calculated by adapting the explicit formulas of Prasad and Rao (1990). The matrix dimension is the number of domains multiplied by the number of categories minus 1.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicator under a multinomial mixed model with correlated time and area effects. Submitted for review.

Prasad, NGN, Rao, JNK (1990). The estimation of the mean squared error of small area estimators. Journal of the American Statistical Association, 85, 163-171.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult.it](#), [prmu.time](#), [phi.direct.it](#), [sPhikf.it](#), [modelfit2](#), [Fbetaf.it](#), [ci](#), [mseb](#)

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=2 #type of model
data(simdata2)
datar=data.mme(simdata2,k,pp,mod)
##Model fit
result=modelfit2(datar$d,datar$t,pp,datar$Xk,datar$X,datar$Z,datar$initial,datar$y[,1:(k-1)],
                datar$n,datar$N)

##Analytic MSE
msef=msef.it(pp,datar$X,result,datar$n,datar$N)
```

 omega

Model correlation matrix for Model 3

Description

This function calculates the model correlation matrix and the first derivative of the model correlation matrix for Model 3. Model 3 is the multinomial mixed model with two independent random effects for each category of the response variable: one domain random effect and another correlated time and domain random effect.

Usage

```
omega(t, k, rho, phi2)
```

Arguments

t	number of time periods.
k	number of categories of the response variable.
rho	vector with the correlation parameter obtained from modelfit3 .
phi2	vector with the values of the second variance component obtained from modelfit3 .

Value

A list containing the following components.

Omega.d	correlation matrix.
First.derivative.Omegad	Fisher derivative of the model correlation matrix.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicator under a multinomial mixed model with correlated time and area effects. Submitted for review.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult.ct](#), [prmu.time](#), [phi.direct.ct](#), [Fbetaf.ct](#), [sPhikf.ct](#), [ci](#), [modelfit3](#), [msef.ct](#), [mseb](#)

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=3 #type of model
data(simdata3) #data
datar=data.mme(simdata3,k,pp,mod)
```

```

initial=datar$initial
mean=prmu.time(datar$n,datar$Xk,initial$beta.0,initial$u1.0,initial$u2.0)

##The model correlation matrix
matrix.corr=omega(datar$t,k,initial$rho.0,initial$phi2.0)

```

phi.direct

Variance components for Model 1

Description

This function calculates the variance components for the multinomial mixed model with one independent random effect in each category of the response variable (Model 1). These values are used in the second part of the fitting algorithm implemented in [modelfit1](#). The algorithm adapts the ideas of Schall (1991) to a multivariate model and the variance components are estimated by the REML method.

Usage

```
phi.direct(sigmoid, phi, X, u)
```

Arguments

sigmoid	a list with the model variance-covariance matrices for each domain obtained from wmatrix .
X	list of matrices with the auxiliary variables obtained from data.mme . The dimension of the list is the number of categories of the response variable minus one.
phi	vector with the initial values of the variance components obtained from modelfit1 .
u	matrix with the values of the random effects obtained from modelfit1 .

Value

a list containing the following components.

phi.new	vector with the variance components.
---------	--------------------------------------

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13, 153-178.

Schall, R (1991). Estimation in generalized linear models with random effects. *Biometrika*, 78, 719-727.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult](#), [prmu](#), [Fbetaf](#), [sPhikf](#), [ci](#), [modelfit1](#), [msef](#), [mseb](#).

Examples

```

k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata) #data
mod=1 #type of model
datar=data.mme(simdata,k,pp,mod)
initial=datar$initial
mean=prmu(datar$n,datar$Xk,initial$beta.0,initial$u.0)
#model variance-covariance matrix
sigmap=wmatrix(datar$n,mean$estimated.probabilities)

##Variance components
phi=phi.direct(sigmap,initial$phi.0,datar$X,initial$u.0)

```

phi.direct.ct

Variance components for Model 3

Description

This function calculates the variance components for the multinomial mixed model with two independent random effects for each category of the response variable: one domain random effect and another correlated time and domain random effect (Model 3). These variance components are used in the second part of the fitting algorithm implemented in `modelfit3`. The algorithm adapts the ideas of Schall (1991) to a multivariate model. The variance components are estimated by the REML method.

Usage

```
phi.direct.ct(p, sigmap, X, theta, phi1, phi2, u1, u2, rho)
```

Arguments

p	vector with the number of auxiliary variables per category.
sigmap	a list with the model variance-covariance matrices for each domain obtained from <code>wmatrix</code> .
X	list of matrices with the auxiliary variables obtained from <code>data.mme</code> . The dimension of the list is the number of categories of the response variable minus one.
theta	matrix with the estimated log-probabilities of each category in front of the reference category obtained from <code>prmu.time</code> .
phi1	vector with the initial values of the first variance component obtained from <code>modelfit3</code> .
phi2	vector with the initial values of the second variance component obtained from <code>modelfit3</code> .
u1	matrix with the values of the first random effect obtained from <code>modelfit3</code> .
u2	matrix with the values of the second random effect obtained from <code>modelfit3</code> .
rho	vector with the initial values of the correlation parameter obtained from <code>modelfit3</code> .

Value

a list containing the following components.

phi1.new vector with the values of the variance component for the first random effect.
 phi2.new vector with the values of the variance component for the second random effect.
 rho.new vector with the correlation parameter.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicator under a multinomial mixed model with correlated time and area effects. Submitted for review.

Schall, R (1991). Estimation in generalized linear models with random effects. *Biometrika*, 78,719-727.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult.ct](#), [prmu.time](#), [Fbetaf.ct](#) [sPhikf.ct](#), [ci](#), [modelfit3](#), [msef.ct](#), [mseb](#), [omega](#)

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=3 #type of model
data(simdata3) #data
datar=data.mme(simdata3,k,pp,mod)
initial=datar$initial
mean=prmu.time(datar$n,datar$Xk,initial$beta.0,initial$u1.0,initial$u2.0)
sigmap=wmatrix(datar$n,mean$estimated.probabilities)

##The variance components
phi.ct=phi.direct.ct(pp,sigmap,datar$X,mean$eta,initial$phi1.0,
  initial$phi2.0,initial$u1.0,initial$u2.0,initial$rho.0)
```

 phi.direct.it

Variance components for Model 2

Description

This function calculates the variance components for the multinomial mixed model with two independent random effects for each category of the response variable: one domain random effect and another independent time and domain random effect (Model 2). This variance components are used in the second part of the fitting algorithm implemented in [modelfit2](#). The algorithm adapts the ideas of Schall (1991) to a multivariate model. The variance components are estimated by the REML method.

Usage

```
phi.direct.it(pp, sigmap, X, phi1, phi2, u1, u2)
```

Arguments

pp	vector with the number of auxiliary variables per category.
sigmap	a list with the model variance-covariance matrices for each domain obtained from wmatrix .
X	list of matrices with the auxiliary variables obtained from data.mme . The dimension of the list is the number of categories of the response variable.
phi1	vector with the initial values of the first variance component obtained from modelfit2 .
phi2	vector with the initial values of the second variance component obtained from modelfit2 .
u1	matrix with the values of the first random effect obtained from modelfit2 .
u2	matrix with the values of the second random effect obtained from modelfit2 .

Value

a list containing the following components.

phi1.new	vector with the values of the variance component for the first random effect.
phi2.new	vector with the values of the variance component for the second random effect.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicators under a multinomial mixed model with correlated time and area effects. Submitted for review.

Schall, R (1991). Estimation in generalized linear models with random effects. *Biometrika*, 78,719-727.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult.it](#), [prmu.time](#), [Fbetaf.it](#) [sPhikf.it](#), [ci](#), [modelfit2](#), [msef.it](#), [mseb](#)

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
d=10 #number of areas
mod=2 #Type of model
data(simdata2) #data
datar=data.mme(simdata2,k,pp,mod)
initial=datar$initial
mean=prmu.time(datar$n,datar$Xk,initial$beta.0,initial$u1.0,initial$u2.0)
sigmap=wmatrix(datar$n,mean$estimated.probabilities) #variance-covariance
```

```
## The variance components
phi.it=phi.direct.it(pp,sigmap,datar$X,initial$phi1.0,initial$phi2.0,
                    initial$u1.0,initial$u2.0)
```

 phi.mult

Initial values for the variance components for Model 1

Description

This function is used in [initial.values](#) to calculate the initial values for the variance components in the multinomial mixed model with one independent random effect in each category of the response variable (Model 1).

Usage

```
phi.mult(beta.0, y, Xk, M)
```

Arguments

beta.0	initial values for the fixed effects obtained in initial.values .
y	matrix with the response variable obtained from data.mme . The rows are the domains and the columns are the categories of the response variable.
Xk	list of matrices with the auxiliary variables per category obtained from data.mme . The dimension of the list is the number of domains.
M	vector with the sample size of the areas.

Value

phi.0 vector of initial values for the variance components

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13, 153-178.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [prmu](#), [Fbetaf](#), [phi.direct](#), [sPhikf](#), [ci](#), [modelfit1](#), [msef](#), [mseb](#).

Examples

```

k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata) #data
mod=1 #type of model
datar=data.mme(simdata,k,pp,mod)
###beta values
beta.new=list()
beta.new[[1]]=matrix(c( 1.3,-1),2,1)
beta.new[[2]]=matrix(c( -1.6,1),2,1)

##Initial variance components
phi=phi.mult(beta.new,datar$y,datar$Xk,datar$n)

```

phi.mult.ct

Initial values for the variance components in Model 3

Description

This function is used in [initial.values](#) to calculate the initial values for the variance components in the multinomial mixed model with two independent random effects for each category of the response variable: one domain random effect and another correlated time and domain random effect (Model 3).

Usage

```
phi.mult.ct(beta.0, y, Xk, M, u1, u2)
```

Arguments

beta.0	a list with the initial values for the fixed effects per category obtained from initial.values .
y	matrix with the response variable obtained from data.mme . The rows are the domains and the columns are the categories of the response variable minus one.
Xk	list of matrices with the auxiliary variables per category obtained from data.mme . The dimension of the list is the number of domains.
M	vector with the sample size of the areas.
u1	matrix with the values for the first random effect obtained from initial.values .
u2	matrix with the values for the second random effect obtained from initial.values .

Value

A list containing the following components.

phi.0	vector of the initial values for the variance components.
rho.0	vector of the initial values for the correlation parameter.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicators under a multinomial mixed model with correlated time and area effects. Submitted for review.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [prmu.time](#), [Fbetaf.ct](#), [phi.direct.ct](#), [sPhikf.ct](#), [ci](#), [modelfit3](#), [msef.ct](#), [omega](#), [mseb](#).

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=3 #type of model
data(simdata3) #data
D=nrow(simdata3)
datar=data.mme(simdata3,k,pp,mod)
###Fixed effects values
beta.new=list()
beta.new[[1]]=matrix(c( 1.3,-1),2,1)
beta.new[[2]]=matrix(c( -1.6,1),2,1)
## Random effects values
u1.new=rep(0.01,((k-1)*datar$d))
dim(u1.new)=c(datar$d,k-1)
u2.new=rep(0.01,((k-1)*D))
dim(u2.new)=c(D,k-1)

## Initial variance components
phi=phi.mult.ct(beta.new,datar$y,datar$Xk,datar$n,u1.new,u2.new)
```

phi.mult.it

Initial values for the variance components in Model 2

Description

This function is used in [initial.values](#) to calculate the initial values for the variance components in the multinomial mixed model with two independent random effects for each category of the response variable: one domain random effect (u1) and another independent time and domain random effect (u2) (Model 2).

Usage

```
phi.mult.it(beta.0, y, Xk, M, u1, u2)
```

Arguments

beta.0	initial values for the fixed effects obtained from initial.values .
y	matrix with the response variable obtained from data.mme . The rows are the domains and the columns are the categories of the response variable.
Xk	list of matrices with the auxiliary variables per category obtained from data.mme . The dimension of the list is the number of domains.
M	vector with the sample size of the areas.
u1	vector with the initial values for the first random effect obtained from initial.values .
u2	vector with the initial values for the second random effect obtained from initial.values .

Value

phi.0 vector of the initial values for the variance components.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13, 153-178.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [prmu.time](#), [Fbetaf.it](#), [phi.direct.it](#), [sPhikf.it](#), [ci](#), [modelfit2](#), [msef.it](#), [mseb](#).

Examples

```

k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata2) #data
mod=2 #Type of model
datar=data.mme(simdata2,k,pp,mod)
D=nrow(simdata2)
###fixed effects values
beta.new=list()
beta.new[[1]]=matrix(c( 1.3,-1),2,1)
beta.new[[2]]=matrix(c( -1.6,1),2,1)
## random effects values
u1.new=rep(0.01,((k-1)*datar$d))
dim(u1.new)=c(datar$d,k-1)
u2.new=rep(0.01,((k-1)*D))
dim(u2.new)=c(D,k-1)

##Initial variance components
phi=phi.mult.it(beta.new,datar$y,datar$Xk,datar$n,u1.new,u2.new)

```

print.mme	<i>Print objects of class mme</i>
-----------	-----------------------------------

Description

This function prints objects of class mme.

Usage

```
## S3 method for class 'mme'
print(x, ...)
```

Arguments

x a list with the output of `modelfit1`, `modelfit2` or `modelfit3`.
 ... further information.

See Also

[modelfit1](#), [modelfit2](#), [modelfit3](#)

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=1 # Type of model
data(simdata)
datar=data.mme(simdata,k,pp,mod)
##Model fit
result=modelfit1(pp,datar$Xk,datar$X,datar$Z,datar$initial,datar$y[,1:(k-1)],datar$n,datar$N)
result
```

prmu	<i>Estimated mean and probabilities for Model 1</i>
------	---

Description

This function calculates the estimated probabilities and the estimated mean of the response variable, in the multinomial mixed model with one independent random effect in each category of the response variable (Model 1).

Usage

```
prmu(M, Xk, beta, u)
```

Arguments

M	vector with the area sample sizes.
Xk	list of matrices with the auxiliary variables per category obtained from data.mme . The dimension of the list is the number of domains.
beta	fixed effects obtained from modelfit1 .
u	values of random effects obtained from modelfit1 .

Value

A list containing the following components:

Estimated.probabilities	matrix with the estimated probabilities for the categories of response variable.
mean	matrix with the estimated mean of the response variable.
eta	matrix with the estimated log-rates of the probabilities of each category over the reference category.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13, 153-178.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult](#), [Fbetaf](#), [phi.direct](#), [sPhikf](#), [ci](#), [modelfit1](#), [msef](#), [mseb](#).

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata) #data
mod=1 #type of model
D=nrow(simdata)
datar=data.mme(simdata,k,pp,mod)
initial=datar$initial

##Estimated mean and probabilities
mean=prmu(datar$n,datar$Xk,initial$beta.0,initial$u.0)
```

`prmu.time`*Estimated mean and probabilities for Model 2 and 3*

Description

This function calculates the estimated probabilities and the estimated mean of the response variable, in the multinomial mixed models with two independent random effects, one random effect associated with the area and the other associated with the time, for each category of the response variable. The first model assumes independent time and domain random effect (Model 2) and the second model assumes correlated time and domain random effect (Model 3).

Usage

```
prmu.time(M, Xk, beta, u1, u2)
```

Arguments

M	vector with the area sample sizes.
Xk	list of matrices with the auxiliary variables per category obtained from data.mme . The dimension of the list is the number of domains.
beta	a list with the values for the fixed effects beta per category obtained from modelfit2 .
u1	a vector with the values of the first random effect obtained from modelfit2 or modelfit3 .
u2	a vector with the values of the second random effect obtained from modelfit2 or modelfit3 .

Value

A list containing the following components:

Estimated.probabilities	matrix with the estimated probabilities for the categories of response variable.
mean	matrix with the estimated mean of the response variable.
eta	matrix with the estimated log-rates of the probabilities of each category over the reference category.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicator under a multinomial mixed model with correlated time and area effects. Submitted for review.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult.it](#), [Fbetaf.it](#), [phi.direct.it](#), [sPhikf.it](#), [ci](#), [modelfit2](#), [msef.it](#), [mseb](#)

Examples

```

k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=2 #Type of model
data(simdata2) # data
datar=data.mme(simdata2,k,pp,mod)
initial=datar$initial

## Estimated mean and estimated probabilities
mean=prmu.time(datar$n,datar$Xk,initial$beta.0,initial$u1.0,initial$u2.0)

```

simdata

Dataset for Model 1

Description

Dataset used by the multinomial mixed effects model with one independent random effect in each category of the response variable (Model 1). This dataset contains 15 small areas. The response variable has three categories. The last is the reference category. The variables are as follows:

Usage

```
simdata
```

Format

A data frame with 15 rows and 9 variables in columns

Details

- area: area indicator.
- Time: time indicator.
- sample: the sample size of each domain.
- Population: the population size of each domain.
- Y1: the first category of the response variable.
- Y2: the second category of the response variable.
- Y3: the third category of the response variable.
- X1: the covariate for the first category of the response variable.
- X2: the covariate for the second category of the response variable.

Examples

```

k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata) #data
mod=1 # type of model
datar=data.mme(simdata,k,pp,mod)
# Model fit
result=model(datar$d,datar$t,pp,datar$Xk,datar$X,datar$Z,datar$initial,datar$y[,1:(k-1)],
             datar$n,datar$N,mod)

#Analytic MSE
mse=msef(pp,datar$X,datar$Z,result,datar$N,datar$n)

B=1 #Bootstrap iterations
ss=12345 #SEED
set.seed(ss)

##Bootstrap parametric BIAS and MSE
mse.pboot=mseb(pp,datar$Xk,datar$X,datar$Z,datar$n,datar$N,result,B,mod)

```

simdata2

Dataset for Model 2

Description

Dataset used by the multinomial mixed effects model with two independent random effects in each category of the response variable: one domain random effect and another independent time and domain random effect (Model 2). This dataset contains 10 small areas and two periods. The response variable has three categories. The last is the reference category. The variables are as follows:

Usage

```
simdata2
```

Format

A data frame with 30 rows and 9 variables in columns

Details

- area: area indicator.
- Time: time indicator.
- sample: the sample size of each domain.
- Population: the population size of each domain.
- Y1: the first category of the response variable.
- Y2: the second category of the response variable.

- Y3: the third category of the response variable.
- X1: the covariate for the first category of the response variable.
- X2: the covariate for the second category of the response variable.

Examples

```

k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata2)
mod=2 #type of model
datar=data.mme(simdata2,k,pp,mod)

##Model fit
result=model(datar$d,datar$t,pp,datar$Xk,datar$X,datar$Z,datar$initial,datar$y[,1:(k-1)],
             datar$n,datar$N,mod)

##Analytic MSE
msef=msef.it(pp,datar$X,result,datar$n,datar$N)

B=1 #Bootstrap iterations
ss=12345 #SEED
set.seed(ss)

##Bootstrap parametric BIAS and MSE
mse.pboot=mseb(pp,datar$Xk,datar$X,datar$Z,datar$n,datar$N,result,B,mod)

```

simdata3

Dataset for Model 3

Description

Dataset used by the multinomial mixed effects model with two independent random effects in each category of the response variable: one domain random effect and another correlated time and domain random effect (Model 3). This dataset contains ten small areas and four periods. The response variable has three categories. The last is the reference category. The variables are as follows:

Usage

```
simdata3
```

Format

A data frame with 40 rows and 9 variables in columns

Details

- area: area indicator.
- Time: time indicator.
- sample: the sample size of each domain.
- Population: the population size of each domain.
- Y1: the first category of the response variable.
- Y2: the second category of the response variable.
- Y3: the third category of the response variable.
- X1: the covariate for the first category of the response variable.
- X2: the covariate for the second category of the response variable.

Examples

```
## Not run:

k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=3 #type of model
data(simdata3) #data
datar=data.mme(simdata3,k,pp,mod)
##Model fit
result=model(datar$d,datar$t,pp,datar$Xk,datar$X,datar$Z,datar$initial,datar$y[,1:(k-1)],
             datar$n,datar$N,mod)

##Analytic MSE
msef=msef.ct(pp,datar$X,result,datar$n,datar$N)

B=1 #Bootstrap iterations
ss=12345 #SEED
set.seed(ss)

##Bootstrap parametric BIAS and MSE
mse.pboot=mseb(pp,datar$Xk,datar$X,datar$Z,datar$n,datar$N,result,B,mod)
## End(Not run)
```

sPhikf

Fisher information matrix and score vectors of the variance components for Model 1

Description

This function computes the Fisher information matrix and the score vectors of the variance components, for the multinomial mixed model with one independent random effect in each category of the response variable (Model 1). These values are used in the fitting algorithm implemented in [modelfit1](#) to estimate the random effects. The algorithm adapts the ideas of Schall (1991) to a multivariate model. The variance components are estimated by the REML method.

Usage

```
sPhikf(pp, sigmap, X, eta, phi)
```

Arguments

pp	vector with the number of the auxiliary variables per category.
sigmap	a list with the model variance-covariance matrices for each domain obtained from wmatrix .
X	list of matrices with the auxiliary variables obtained from data.mme . The dimension of the list is the number of categories of the response variable minus one.
eta	matrix with the estimated log-rates of probabilities of each category over the reference category obtained from prmu .
phi	vector with the values of the variance components obtained from modelfit1 .

Value

A list containing the following components.

S.k	phi score vector.
F	Fisher information matrix of the variance component phi.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*, 13, 153-178.

Schall, R (1991). Estimation in generalized linear models with random effects. *Biometrika*, 78, 719-727.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult](#), [prmu](#), [phi.direct](#), [Fbetaf](#), [ci](#), [modelfit1](#), [msef](#), [mseb](#).

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
data(simdata) #data
mod=1 #type of model
datar=data.mme(simdata,k,pp, mod)
initial=datar$initial
mean=prmu(datar$n,datar$Xk,initial$beta.0,initial$u.0)
sigmap=wmatrix(datar$n,mean$estimated.probabilities)

##Fisher information matrix and score vectors
Fisher.phi=sPhikf(pp,sigmap,datar$X,mean$eta,initial$phi.0)
```

sPhikf.ct

Fisher information matrix and score vectors of the variance components for Model 3

Description

This function computes the Fisher information matrix and the score vectors of the variance components, for the multinomial mixed model with two independent random effects for each category of the response variable: one domain random effect and another correlated time and domain random effect (Model 3). These values are used in the fitting algorithm implemented in `modelfit3` to estimate the random effects. The algorithm adapts the ideas of Schall (1991) to a multivariate model. The variance components are estimated by the REML method.

Usage

```
sPhikf.ct(d, t, pp, sigmap, X, eta, phi1, phi2, rho, pr, M)
```

Arguments

d	number of areas.
t	number of time periods.
pp	vector with the number of the auxiliary variables per category.
sigmap	a list with the model variance-covariance matrices for each domain obtained from <code>wmatrix</code> .
X	list of matrices with the auxiliary variables obtained from <code>data.mme</code> . The dimension of the list is the number of categories of the response variable minus one.
eta	matrix with the estimated log-rates of probabilities of each category over the reference category obtained from <code>prmu.time</code> .
phi1	vector with the values of the first variance component obtained from <code>modelfit3</code> .
phi2	vector with the values of the second variance component obtained from <code>modelfit3</code> .
rho	vector with the correlation parameter obtained from <code>modelfit3</code> .
pr	matrix with the estimated probabilities of the response variable obtained from <code>prmu.time</code> .
M	vector with the area sample sizes.

Value

A list containing the following components.

S	(phi1, phi2, rho) score vector.
F	Fisher information matrix of the variance components (phi1, phi2, rho).

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicators under a multinomial mixed model with correlated time and area effects. Submitted for review.

Schall, R (1991). Estimation in generalized linear models with random effects. *Biometrika*, 78,719-727.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult.ct](#), [prmu.time](#), [phi.direct.ct](#), [Fbetaf.ct](#), [omega.ci](#), [modelfit3](#), [msef.ct](#), [mseb](#)

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=3 #type of model
data(simdata3) #data
datar=data.mme(simdata3,k,pp, mod)
initial=datar$initial
mean=prmu.time(datar$n,datar$Xk,initial$beta.0,initial$u1.0,initial$u2.0)
sigmap=wmatrix(datar$n,mean$estimated.probabilities)

## Fisher information matrix and the score vectors
Fisher.phi.ct=sPhikf.ct(datar$d,datar$t,pp,sigmap,datar$X,mean$eta,initial$phi1.0,
    initial$phi2.0,initial$rho.0,mean$estimated.probabilities,datar$n)
```

sPhikf.it

Fisher information matrix and score vectors of the variance components for Model 2

Description

This function computes the Fisher information matrix and the score vectors of the variance components, for the multinomial mixed model with two independent random effects for each category of the response variable: one domain random effect and another independent time and domain random effect (Model 2). These values are used in the fitting algorithm implemented in [modelfit2](#) to estimate the random effects. The algorithm adapts the ideas of Schall (1991) to a multivariate model. The variance components are estimated by the REML method.

Usage

```
sPhikf.it(d, t, pp, sigmap, X, eta, phi1, phi2)
```

Arguments

d	number of areas.
t	number of time periods.
pp	vector with the number of the auxiliary variables per category.
sigmap	a list with the model variance-covariance matrices for each domain obtained from wmatrix .
X	list of matrices with the auxiliary variables obtained from data.mme . The dimension of the list is the number of categories of the response variable minus one.
eta	matrix with the estimated log-rates of probabilities of each category over the reference category obtained from prmu.time .
phi1	vector with the values of the first variance component obtained from modelfit2 .
phi2	vector with the values of the second variance component obtained from modelfit2 .

Value

A list containing the following components.

S	phi score vector.
F	Fisher information matrix of the variance components phi1 and phi2.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Small area estimation of labour force indicators under a multinomial mixed model with correlated time and area effects. Submitted for review.

Schall, R (1991). Estimation in generalized linear models with random effects. *Biometrika*, 78,719-727.

See Also

[data.mme](#), [initial.values](#), [wmatrix](#), [phi.mult.it](#), [prmu.time](#), [phi.direct.it](#), [Fbetaf.it](#), [ci](#), [modelfit2](#), [msef.it](#), [mseb](#)

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=2 #Type of model
data(simdata2) #data
datar=data.mme(simdata2,k,pp,mod)
initial=datar$initial
mean=prmu.time(datar$n,datar$Xk,initial$beta.0,initial$u1.0,initial$u2.0)
sigmap=wmatrix(datar$n,mean$estimated.probabilities)

##Fisher information matrix and score vectors
Fisher.phi=sPhikf.it(datar$d,datar$t,pp,sigmap,datar$X,mean$eta,initial$phi1.0,
                    initial$phi2.0)
```

wmatrix

Model variance-covariance matrix of the multinomial mixed models

Description

This function calculates the variance-covariance matrix of the multinomial mixed models. Three types of multinomial mixed model are considered. The first model (Model 1), with one random effect in each category of the response variable; Model 2, introducing independent time effect; Model 3, introducing correlated time effect.

Usage

```
wmatrix(M, pr)
```

Arguments

M	vector with area sample sizes.
pr	matrix with the estimated probabilities for the categories of the response variable obtained from prmu or prmu.time .

Value

W a list with the model variance-covariance matrices for each domain.

References

Lopez-Vizcaino, ME, Lombardia, MJ and Morales, D (2013). Multinomial-based small area estimation of labour force indicators. *Statistical Modelling*,13,153-178.

See Also

[data.mme](#), [initial.values](#), [phi.mult](#), [prmu](#), [prmu.time](#) [Fbetaf](#), [phi.direct](#), [sPhikf](#), [ci](#), [modelfit1](#), [msef](#), [mseb](#)

Examples

```
k=3 #number of categories of the response variable
pp=c(1,1) #vector with the number of auxiliary variables in each category
mod=2 #type of model
data(simdata2)
datar=data.mme(simdata2,k,pp,mod)
initial=datar$initial
mean=prmu.time(datar$n,datar$Xk,initial$beta.0,initial$u1.0,initial$u2.0)
##The model variance-covariance matrix
varcov=wmatrix(datar$n,mean$estimated.probabilities)
```

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